LEACHATE TRANSPORT MODELING

By

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CHAPTER I

INTRODUCTION

Background Statement

Sanitary landfills are now a widely utilized method for the disposal of solid waste. It is estimated that more than 20,000 landfills are currently being used in the United States to dispose of more than 90% of the municipal and industrial solid wastes (Freeze and Cherry, 1979). Because of low operating and capital costs, simple and flexible operation, and an ability to accommodate all types of materials without need for separate collection, sanitary landfills will continue to be the primary method of disposal of solid waste, at least during the next few decades. However, in recent years, serious problems have been raised about the potential effects of leachate contamination on groundwater quality. Concomitant with increased usage and public awareness is the concern over the potential pollution of leachate emanating from landfills.

Leachate is water which has percolated through solid waste carrying with it soluble and suspended substances. Leachate from a sanitary landfill often contains a high concentration of organic matter and inorganic ions, including heavy metals. The principal source of leachate in the landfill is from precipitation. When the water infiltrates through the surface and exceeds the field capacity (defined as the maximum moisture a soil can hold against the pull of gravitational force) of the cover soil, it percolates down into the refuse. The addition of moisture to refuse over a period of time saturates the refuse to its field-capacity moisture content. At that point, the moisture from the refuse percolates through the lining into the virgin ground below in the form of leachate. The percolation will continue until it reaches an impermeable layer. Then the vertical movement will turn into horizontal migration if sufficient leachate is generated. This may bring a serious problem because the leachate becomes a potential source of water contamination if it joins a surface-water or groundwater source before it is sufficiently attenuated of its impurities. Thus, it is essential to predict the movement of leachate in the subsurface environment.

The successful location and operation of a sanitary landfill require quantitative knowledge of how leachate-contaminant will migrate through the soil-water system. During the past years, much work has been done for the movement of soluble matter in porous media based on theoretical background. There is apparent agreement in the literature on the validity of mathematical equations to model solute transport in porous media. In spite of the great research effort, however, little work has been done to determine the fate of leachate in soil and to model its movement in the soil-water system. The objective herein is to develop a mathematical model that can reasonably represent the leachate transport process and predict its concentration profile as a function of time and space.

A mathematical model in the form of a second-order partial differential equation together with a set of boundary and initial conditions to describe the leachate transport phenomenon is constructed. The governing transport equation includes simultaneous hydrodynamic dispersion, molecular diffusion, flow convection, linear equilibrium adsorption, and first-order transformation processes of leachate movement in soil media.

Three numerical approximations--centered-, backward-, and forward-in-space--based on the Crank-Nicolson finite difference method are derived to solve the mathematical equation. The system of difference equations is solved on an IBM-3081K computer based on simplified Gauss elimination technique for tridiagonal matrices. The performance of the numerical model is compared with an analytical model.

Study Objectives

The purpose of this study is to estimate the spatial and temporal variations of leachate solute concentrations for determining the impact of leachate substances on groundwater quality, and to determine the need for and degree of environmental control. The objectives are as follows:

1. To develop a numerical model of leachate transport below a sanitary landfill or a waste-disposal site.

2. To test the model with an analytical model.

3. To apply the model to a sanitary landfill in order to predict the leachate spatial and temporal concentration distributions under hypothetical conditions.

The model is based on a one-dimensional flow condition. It can be applied to single- or multi-layered soil media under saturated or unsaturated conditions. Each soil layer is considered to be homogeneous and isotropic. The model may also be applied to accommodate both leachate vertical and horizontal movements provided the flow condition and concentration gradients are continuous.

CHAPTER II

LITERATURE REVIEW

Slichter (1905) was among the first to discuss that dispersion-diffusion affects the transport of substances through the porous media. However, quantitative descriptions were not presented until the 1950's by Lapidus and Amundson and others. Lapidus and Amundson (1952) provided analytical solutions for the dispersion equation subject to a linear adsorption and for the case in which nonlinear adsorption is considered, but dispersion is neglected.

A system of partial differential equations describing miscible displacement of fluids in porous media was derived by Peaceman and Rachford (1962). The system includes coupled flow (pressure) and transport (concentration) equations and boundary and initial conditions. The system of differential equations was solved by using the backward-in-space, the centered-in-space, and the combination of both finite difference approximations. The solution of the backward-in-space equation causes numerical smearing (numerical dispersion) and has a first-order accuracy in both space and time, while the solution of the centered-in-space equation produces overshoot and oscillation has a first-order accuracy in time but a second-order accuracy in space. The combination of both difference equations is a modified procedure which improves the accuracy in the neighborhood immediately behind the concentration front. This modification is referred to as "transfer of overshoot."

Banks and Ali (1964) presented a mathamatical analysis of simultaneous dispersion and adsorption of a solute within a porous medium in a one-dimensional steady-flow field subject to a constant input concentration. Analytical solutions were presented for the cases of convection with dispersion but without adsorption, convection with adsorption but without dispersion, and convection with both dispersion and adsorption. In all cases, both equilibrium and nonequilibrium relations between the liquid and solid phases were studied.

Gander et al. (1964) attributed the cause of the numerical dispersion to the fact that, when velocity is large compared to the dispersion coefficient, the parabolic-type transport equation behaves like a hyperbolic-type equation. They proposed a numerical method based on the method of characteristics for the solution of miscible displacement problems. Extensive tests in one-dimensional problems show that the method is accurate over a wide range of values of the dispersion coefficient, including zero. In this method, the partial differential equation is first replaced by characteristic equations, a set of ordinary differential equations, and then these equations are solved by using finite difference approximations. Although the method of characteristics was successful in eliminating numerical dispersion and compared well with the exact solution, the method is difficult to program and large computer storage is required.

Price et al. (1968) proposed a numerical formulation of high-order accuracy, based on variational methods, for the solution of diffusion-convection-type equations. A variable interpolation-function procedure was introduced. This approach allows high-order accuracy in the neighborhood of concentration fronts and low-order accuracy in regions where the solution is smoother, keeping the overall dimensionality of the system small. The results were compared with several finite difference approximations and with a technique based on the method of characteristics.

Gershon and Nir (1969) studied the effects of boundary conditions on the distribution of the tracer in a transport model. The effects of hydrodynamic dispersion, molecular diffusion, radioactive decay, and chemical interactions of the tracer and soil media are also included. The results from analytical and approximative solutions show that in most practical conditions the steady-state experiments are only influenced up to 0.5% and the nonsteady-state experiments are influenced up to 5% in the region of $C/C_o = 0.5$, where C/C_o is the ratio of the measured concentration to the source concentration.

Qasim and Burchinal (1970) conducted an experiment to determine leaching of chloride during the vertically downward movement of water from simulated landfills. Families of curves were established from the experimental results. These curves were directly applied to estimate the concentrations of various components in the leachate from field sanitary landfills. They also presented a generalized method for theoretical determination of the concentrations of some easily extractable materials leached from sanitary landfills. A number of tests were carried out for acidity, alkalinity, BOD, calcium, total iron, total hardness, magnesium, ammonia nitrogen, organic nitrogen, total phosphates, potassium, sodium, total solids, volatile solids, sulfate, tannin, and lignin. Average ratios of these experimental leaching materials to theoretical chloride concentrations were obtained, and the chemical characters of the leachate were established. Experimental and theoretical concentrations of various leaching materials in this study show maximum deviations of 30%. These deviations were attributed largely to the heterogeneous nature of the refuse, microbial activity, and to the state of decomposition.

Chaudhari (1971) used a high-order difference scheme for the system Peaceman and Rachford (1962) used to eliminate the numerical smearing. The technique involves an addition of a negative dispersion term to the transport equation. This additional dispersion term, which is called the numerical dispersion coefficient, accounts for most of the numerical smearing in the numerical solution of the transport equation. The difference analog of the pressure or flow equation is solved implicitly for the pressure distribution, while the difference analog of the solute concentration or transport equation is solved explicitly for the concentration distribution. The flow equation is solved first for an instantaneous pressure distribution and flow velocities. Then, the flow velocities are substituted in the transport equation and solved for a new concentration distribution. The cycle is repeated for each time step. This high-order difference scheme eliminates most of the numerical smearing, leaving only the effect of physical dispersion.

Lantz (1971) quantified the truncation error of numerical dispersion by using the implicit and explicit finite difference approximations developed from Taylor series expansions for miscible and immiscible convective-diffusion equations. He found that the magnitude of the numerical dispersion or diffusion can depend on both space and time step-sizes. The effect of numerical dispersion, then, can be minimized by choosing adequate step-sizes. He also found that the numerical dispersion for the implicit backward or central difference scheme is always greater than for the equivalent explicit scheme. However, the implicit method is always stable for both backward and central difference schemes.

The simultaneous transfer of solute and water during infiltration through an unsaturated soil was studied by Warrick et al. (1971), both in the field and numerically. The field results show that the displacement of chloride applied in irrigation water and leached with additional chloride-free water can be quantitatively predicted by linking the equations of solute and water movement through an unsaturated soil. The value of the dispersion coefficient is found at least one or two orders of magnitude larger than molecular diffusion. The solute movement is shown to be nearly independent of the initial moisture content but highly dependent on the infiltration rate and moisture content maintained at the soil surface during the infiltration.

Freeze (1972) developed a two-dimensional model to simulate the effects of recharge from deep-well injection, waste-disposal ponds, and sanitary landfills on the groundwater flow system. The model predicts only convective transport and does not consider dispersion or hydrochemical interactions between pollutants and soils. The model can be applied at the reconnaissance stage on a regional basis to analyze the suitability of a large number of potential disposal sites. Quantitative interpretation of the output provides predictive values of the rate of pollutants into the flow system, lengths of flow paths, travel times of pollutants, water-table movements, and discharge rates to surface water.

Lai and Jurinak (1972) provided numerical solutions of the dispersion equation for different adsorption equilibriums by using the explicit finite difference scheme. The scheme is restricted by small grid spacings in order to insure stability in the computations. The scheme may be subject to significant truncation error, since a first-order correct approximation is used for the time derivative.

A detailed example of applying modeling techniques for the movement of chemicals through soil media was discussed by Boast (1973). Based on the classical mathematical macroscopic continuum theories, some models and equations for describing the conservation of mass, hydrodynamic dispersion, molecular diffusion, convection, adsorption or exchange, equilibrium isotherm, and source or sink were presented in tabular form. Each component was discussed in detail for various modeling techniques.

Bresler (1973) applied a mathematical model for simulating the transport of noninteracting solutes and water in unsaturated soils during non-steady-state infiltration, redistribution, and evaporation. The combined effects of convection, molecular diffusion, and mechanical dispersion are investigated and analyzed. The transient transport equation is solved numerically by the implicit finite difference procedure that eliminates most of the numerical dispersion that may arise from the numerical solution. An expression for the first space derivative in the governing partial differential equation is developed with the aid of Taylor series expansion in a manner similar to that employed by Chaudhari (1971). It is believed that the numerical dispersion stems primarily from the numerical approximations of the first-order time and space derivatives.

Gupta and Greenkorn (1973) used the Crank-Nicolson finite difference method, which is the average of the explicit and implicit methods, to solve the dispersion-convection equation subject to a bilinear rate of adsorption. The method utilizes second-order correct analogs for both the space and time derivatives. All of the finite differences were written about the point halfway between the known and unknown time levels. The solutions for the two coupled nonlinear parabolic partial differential equations were presented for a range of variables covering the practical values of flow velocity, disperson coefficient, and kinetic rate constant that were involved in the movement of nitrate and phosphate ions in porous media.

Kirda et al. (1973) studied the displacement of chloride during infiltration using soil columns for two cases: chloride initially spread on the soil surface and choloride initially mixed with the soil. Chloride was applied as $CaCl_2$. Two equations describing the vertical water flow and the miscible displacement of chloride were solved simultaneously by using the explicit finite difference analog. Treatments include different values of initial soil-water content and surface-water content during infiltration. The results were tested with the experimental data.

Schwartz and Domenico (1973) conducted an investigation on the effects of physical, chemical, and biological processes in a simulation model that incorporates mass transfer rates and reaction kinetics. Quantitative analysis with the model indicates that mineral dissolution, precipitation, ion exchange and reactions, saturation constraints, partial pressure of CO_2 gas, shift of the equilibrium concentration, and duration of the processes acting in relation to the residence time of the groundwater flow all play different roles in determining the spatial distribution of the ionic constituents. The utility of the model was demonstrated by applying it to the groundwater reservoir in the Upper Kettle Creek watershed in Ontario, Canada. The results indicate that, although the processes considered within the framework of the model are idealized and lose some of the flexibility and sensitivity of the natural processes, a satisfactory correspondence of the real and idealized systems can be achieved by a trial and error procedure.

Marino (1974a) analyzed a mathematical model for simultaneous dispersion and adsorption of a solute within homogeneous and isotropic porous media in steady-state unidirectional flow fields. The dispersion system was considered to be adsorbing the solute at a rate proportional to its concentration and was subject to input concentrations that vary exponentially with time. The expression take into account the decay of a radioactive contaminant as well as mass transfer from the liquid to the solid phase due to adsorption. Marino (1974b) also presented numerical and analytical solutions for the distribution of a contaminant within a finite, adsorbing, porous medium in a unidirectional flow field. The adsorbing medium was assumed to be homogeneous and isotropic and to act as a mathematical sink. The source concentration was considered to be a step-function of time. The solutions predict the concentration of contaminants as a function of time and space if seepage flow and dispersion and adsorption coefficients are prescribed. Two analytical solutions for two longitudinal dispersion problems in idealized, nonadsorbing, one-dimensional, steady-state, saturated, homogeneous, and isotropic porous media were also derived by Marino (1974c). The seepage flows were assumed to be unidirectional through semi-infinite porous media, and the average seepage velocities were taken to be constant throughout the flow fields. The solutions predict the distribution of contaminants resulting from the variable source concentrations.

Perez et al. (1974) studied both water flow and quality processes occurring on the ground surface, in the unsaturated soil zone, and in the saturated or groundwater zone. The objective was to improve already available formulations for these processes and, subsequently, to develop a methodology for interfacing the individual models. Emphasis was placed on the modeling of agricultural pollution. Nitrogen and phosphorous were the main substances considered. They selected the Lake Apopka Basin in central Florida as the study area to test the interfacing model. The data base for this basin was limited. The limitation of data precludes a complete verification of the model. The results indicate that it is feasible to model a conjunctive surface-ground water system. However, much remains to be accomplished in the usage and interpretation of agricultural information affecting water quality, such as soil erosion, nutrient reactions in the soil, etc.

Tagamets and Sternberg (1974) formulated a one-dimensional convection-dispersion equation subject to a nonlinear adsorption isotherm and solved by using a predictor-corrector finite difference scheme. The effects of convection, dispersion, and adsorption were examined for some typical values of variables. The numerical error in the predictor-corrector method becomes amplified as a result leading to oscillations in the solution for cases in which the dispersion coefficient is small in comparison with the seepage velocity.

Independently measured soil and soil-pesticide adsorption-desorption characteristics to describe the movement of pesticides in a soil profile were solved numerically by Davidson et al. (1975). The implicit finite difference approximations were used for both the water flow and solute transport equations. Numerical dispersion in the finite difference solution of the solute transport equation was considered and, by using the Taylor series expansion, a correction included in the solution. The solution with a correction for numerical dispersion was within 5% of the analytical solution. The kinetic adsorption-desorption models were evaluated and found inadequate for predicting herbicide mobility at high average pore-water velocities. Based upon experimental data, an empirical model was also developed to describe the herbicide movement at high-flow rates.

Laumbach (1975) developed a so-called truncation cancellation procedure (TCP) to improve the accuracy of numerical solution of the convection-diffusion equation. The rationale underlying the treatment is to cancel a portion of the error in the convection term. The application of this technique results in a new finite difference representation of the equation that is accurate to the fourth order when the convection is strongly predominant and the equation acts like a hyperbolic-type equation. Comparison of the results with conventional numerical techniques, exact analytical solutions, and other high-order accuracy numerical methods presented by Garder et al. (1964), Price et al. (1968), Lantz (1971), and Chaudhari (1971), proved that the TCP method has an accuracy superior to other formulations based on two time levels and three spatial locations.

A two-dimensional finite element model was used by Cabrera and Marino (1976) to simulate the transport and distribution of a conservative substance in a stream-aquifer system. The solutions are obtained by solving sequentially the groundwater flow and mass transport equations. A variational approach in conjunction with the finite element method is used to solve the groundwater flow equation. Galerkin's approach coupled with the finite element method is used to solve the mass transport equation. Linear approximated triangular elements and a centered scheme of numerical integration are employed to calculate the hydraulic head distribution and the concentration of solute in the flow region. The linear approximation used to define the concentration function within each element is not appropriate for cases involving steep concentration gradients. For such cases, higher-order approximations are necessary to assure the continuity of gradients across interelemental boundaries. The model was applied to the movement of contaminants in stream-aquifer systems receiving deep percolation.

Ion exchange can be one of the controlling reactions in the flow of solutes through porous media. It has been successfully modeled using both equilibrium- and rate-controlled reactions by Grove (1976). The rate-controlled model is found to be dominated by external or internal diffusion with the actual exchange reaction assumed to be very rapid. However, the equilibrium-controlled model is found to be simpler to use than the rate-controlled model and sometimes sufficient to describe the ion exchange process. The effect of radioactive decay is represented by a first-order irreversible decay reaction and can be easily incorporated into the transport equation.

A mathematical model was simulated by Selim et al. (1976) to describe potassium reactions and transport in soils. Potassium was considered to be present in the soil in four phases: solution, exchangeable, nonexchangeable, and primary mineral. First-order kinetic reactions were assumed to govern the transformation between these four phases. The effect of kinetic rate coefficients on transport and transformation of applied potassium was also investigated. The results of potassium distribution illustrate the dependence of the leaching loss and transformation among the various potassium phases upon the rate coefficients governing the transformation mechanisms.

A general equation describing the three-dimensional transport and dispersion of a dissolved chemical reacting in flowing groundwater was derived by Konikow and Grove (1977), based on the principle of mass conservation. This general solute transport equation relates concentration changes to hydrodynamic dispersion, convective transport, fluid sources and sinks, and chemical reactions. The dispersion coefficient was assumed to be related to the dispersivity of the porous media and to the flow velocity of the groundwater. Because both the dispersion and convection terms in the solute transport equation depend on the flow velocity, the solute transport equation was solved in conjunction with the groundwater flow equation. However, under saturated, homogeneous, and steady-state conditions, the solution of these equations can be considerably simplified. For a flow with a constant velocity, the flow equation is omitted.

Selim et al. (1977) studied a solute transport model for multilayered soils based on laboratory experiments and finite difference approximations to the solute transport equation. The objective of the study was to describe the transport of reactive solutes through water-saturated and unsaturated multilayered soils. Each soil layer was considered homogeneous and isotropic with soil-water and solute adsorption properties known. Linear and nonlinear equilibrium adsorption and first-order kinetic adsorption processes were used to describe solute adsorption in each soil layer. Experimental and calculated results were presented for the movement of chloride and herbicide in two-layered soils where each soil layer possessed specific soil-water and solute adsorption characteristics. The results showed that for a water-saturated multilayered column regardless of soil-water and solute characteristics, the order of soil layering did not influence the effluent concentration distribution. For unsaturated multilayered profiles, the results showed that the use of average water content for each soil layer provided identical effluent concentration distributions to those obtained where actual water content distributions were used. The results establish that the problem of solute transport through water-unsaturated multilayered soil profiles can be significantly simplified.

Wierenga (1977) compared two numerical models for simultaneous movement of water and salts in soil profiles. In the first model, a steady-state model, the water content and flux were kept constant during irrigation with time and depth. In the second model, a transient or unsteady-state model, the water contents and fluxes were varied according to the hydraulic properties following each infiltration. An IBM Continuous System Modeling Program (CSMP) solution method, equivalent to an explicit finite difference scheme, was used. From the comparison between concentration distributions obtained with the two models, Wierenga concluded that for predicting the quality of drainage water, the use of a steady-state model can be justified. His conclusion was further confirmed by experimental data. He also found that the computer time can be from 10 to 100 times more, depending on the conditions and the problem at hand, when a transient model is used as compared to a steady-state model.

Analytical solutions for one-, two-, and three-dimensional solute transport equations, either in closed form or integral form, have been provided by Cleary and Ungs (1978). Analytical solutions for transport with linear equilibrium adsorption, first-order decay or transformation, and zero-order production also have been obtained by Van Genuchten and Alves (1982).

Konikow and Bredehoeft (1978) presented a mathematical model which coupled the groundwater flow equation with the solute transport equation. They used an alternating direction implicit procedure (ADIP) to solve a finite difference approximation to the groundwater flow equation, and they used the method of characteristics (MOC) to solve the solute transport equation. The latter involves a particle tracking procedure to represent convective transport and a two-step explicit procedure to solve a finite difference equation that describes the changes in concentration caused by hydrodynamic dispersion, fluid sources and sinks, and flow velocity. The accuracy of the model was evaluated for two idealized problems for which analytical solutions could be obtained.

Gupta and Singh (1980) conducted analytical solutions to the dispersion-convection equation for leaching of saline water under exponentially decreasing and arbitrary initial salt distribution. The effect of initial condition and solute transport parameters on the predictive behavior of the model was studied. The results indicate that the accuracy of the model increases if the initial salt distribution profile is represented by a function which approximates this profile closely. The effect of the time-varying boundary condition is negligible for most practical purposes at or below 15 cm depth of the soil profile. The changes in the transport parameters may not only enhance or retard the pace of reclamation but also affect the final salt distribution in the soil profile.

A two-dimensional mathematical model for the migration of groundwater contamination was developed by Gureghian et al. (1980). The finite element method using isoparametric elements, based on the Galerkin formulation and on weighting functions of nonsymmetric form, was used to formulate the numerical description of the convective-dispersive mass transport equation. A comparison of two solution schemes with a two-dimensional analytical solution was presented. A field application of the model to the leachate migration from the Babylon sanitary landfill in

Long Island City, New York, illustrating the calibration and verification of the transport parameters, was presented by Gureghian et al. (1981). The transport parameters derived in the calibration period provided good agreement with the observed field concentrations in the verification period.

Satter et al. (1980) used a mathematical model to simulate chemical transport phenomena in porous media, considering Langmuir equilibrium adsorption as well as Langmuir rate-controlled (time-dependent) adsorption. Parametric studies were made using the numerical model to demonstrate the effects of dimensionless dispersion, adsorptive capacity, flow rate, and kinetic rate on chemical transport behavior in a porous medium. The accuracy of the numerical model was verified by comparing the calculated results with those obtained by analytical solutions for a number of cases.

Prakash (1982) applied some simple analytical models to predict the spatial distribution of steady-state concentrations caused by continuous release of contaminants from a point, line, or plane source in a groundwater environment with one- or two-dimensional uniform flow. The models can be used to approximate transports through stream-aquifer systems and through confined or unconfined aquifers. Examples were provided to illustrate the effect of source configuration, such as point, line, or plane source, and the presence or absence of a fully penetrating perennial stream in the flow field. The results of the analytical model were compared with those obtained from a finite element study. The analytical model was proved to be simple, to require less computer time, and to be free from convergence and stability problems associated with some numerical models.

Straub and Lynch (1982a) developed mathematical models for the movement of inorganic contaminants and moisture in sanitary landfills. The models were based on simple well-mixed reactor concepts and on unsaturated flow and transport in porous media. Computer simulations were obtained for laboratory scale experimental landfills. Comparison of simulated and observed results indicates that leachate behavior is explainable in terms of fundamental transport processes. It was found that the roles of moisture retention in the landfill and dilution by infiltrating water are important in predicting leachate quantity and quality. Straub and Lynch (1982b) also applied models to the dissolution, transport, and decay of organic substances in unsaturated sanitary landfills. The models were based on simple well-mixed reactor and vertically cascaded reactor concepts, and on unsaturated moisture flow and contaminant transport in porous media. The roles of aerobic and anaerobic microbial activities were simulated using conventional kinetic formulations. Results indicate that aerobic activity is negligible over the leaching life of a landfill, and that convective removal and anaerobic microbial activity dominate the leachate organic stabilization.

A model for predicting the concentration of leachate organics, measured as chemical oxygen demand (COD), in groundwater below sanitary landfill sites was constructed by Sykes et al. (1982). Simultaneous substrate utilization and microbial mass production equations, with convection and dispersion included for the former, were used for the modeling of leachate organics transport. Biochemical degradation and adsorption were considered as the governing organic matter concentration attenuation mechanisms during transport through soils. The results indicated that substantial removal of leachate organics can be expected within a short distance from the landfill bed.

The finite difference method is the most popular numerical method that was used in solving the mathematical equations. The basis of the finite difference method as a general tool for approximations of flow and transport equations has been emphasized here. The main reasons for the continuous use of the finite difference method can be summarized as follows:

1. Ease in understanding the theoretical basis.
2. Less effort in programming and preparing the data input.
3. High efficiency and low cost in finding the solutions.

CHAPTER III

MODEL FORMULATION

Governing Transport Equation

The movement of leachate substances in soils can be described quantitatively by the law of mass conservation or the continuity equation. For a one-dimensional transport in the x-direction, the continuity equation can be expressed as

$$\frac{\partial(\theta C)}{\partial t} + \sum_i \frac{\partial F_i}{\partial x} + \sum_j Q_j = 0 \tag{3.1}$$

and using the mass (M), length (L), and time (T) system, where

x = space coordinate, L;

t = time coordinate, T;

θ = volumetric moisture content, $L^3\,L^{-3}$;

C = concentration of substance in leachate solution, $M\,L^{-3}$;

F_i = rate of substance movement, $M\,L^{-2}\,T^{-1}$; and

Q_j = rate of substance loss, $M\,L^{-3}\,T^{-1}$.

The three primary mechanisms involved in the movement of leachate substances in soils are: (1) hydrodynamic dispersion due to mechanical mixing, (2) molecular diffusion due to concentration gradients, and (3) convection due to mass flow of the leachate. The mechanical mixing is the result of velocity variations within the porous media. The hydrodynamic dispersion, infrequently referred to as mechanical dispersion, and the molecular diffusion, infrequently referred to as self-diffusion, are represented in the same form of equations; and their effects on transport can be added together:

$$F_h = -D_h \frac{\partial(\theta C)}{\partial x} \tag{3.2}$$

$$F_m = -D_m \frac{\partial(\theta C)}{\partial x} \tag{3.3}$$

$$F_1 \doteq -(D_h + D_m) \frac{\partial(\theta C)}{\partial x} = -D \frac{\partial(\theta C)}{\partial x} \tag{3.4}$$

where

D_h = hydrodynamic dispersion coefficient, $L^2 T^{-1}$;

D_m = molecular diffusion coefficient, $L^2 T^{-1}$;

D = apparent dispersion or dispersion-diffusion coefficient, $L^2 T^{-1}$;

F_h = rate of substance movement by hydrodynamic dispersion, $M L^{-2} T^{-1}$;

F_m = rate of substance movement by molecular diffusion, $M L^{-2} T^{-1}$; and

F_1 = rate of substance movement by hydrodynamic dispersion and molecular diffusion, $M L^{-2} T^{-1}$.

The hydrodynamic dispersion cannot occur without the movement of leachate in porous media. It is related to the pore-water or seepage velocity, a velocity that is equal to the average flow velocity divided by the volumetric moisture content. The molecular diffusion is also velocity dependent (Nielson and Biggar, 1963). However, molecular diffusion occurs even under a no-movement condition. When the solution is moving, molecular diffusion and hydrodynamic dispersion are mechanisms that cause mixing of ionic or molecular constituents. Molecular diffusion stops only when the concentration gradient becomes zero. The molecular diffusion coefficient is temperature dependent, and for different ionic species it may differ. For a combination of ions, it is the average of the separate coefficients (Gardner, 1965). The combination of hydrodynamic dispersion and molecular diffusion is the apparent dispersion or dispersion-diffusion term in the governing transport equation.

Convection is the transport of a substance with the solution. The processes of convection and hydrodynamic dispersion are two inseparable but logically distinct process (Boast, 1973). Disregarding any interaction effects, the convection of a substance by the movement of leachate is given as

$$F_2 = V\theta C \tag{3.5}$$

where V is the average pore water or seepage velocity, $L T^{-1}$; and F_2 is the rate of substance movement by convection, $M L^{-2} T^{-1}$.

Two processes, adsorption and transformation, are considered in this study to account for the loss of leachate substance. Kay and Elrick (1967) and Davidson and Chang (1972) have shown that for many chemicals the equilibrium relationship between the amount of substance adsorbed on the soil surface (S) and the substance concentration in the solution (C) can be described by the Freundlich equilibrium adsorption isotherm

$$S = KC^n \tag{3.6}$$

where n is a constant and for most chemicals $1 \leq n \leq 1.4$. An equilibrium adsorption state with a linear relationship between a substance in the leachate solution and adsorbed on the soil surface is assumed, i.e., $n = 1$. The rate of substance loss by adsorption is then

$$Q_1 = \rho \frac{\partial S}{\partial t} = \rho K \frac{\partial C}{\partial t} \tag{3.7}$$

where

K = distribution coefficient, $L^3\,M^{-1}$;

ρ = bulk density of dry soil, $M\,L^{-3}$;

S = amount of substance adsorbed on the soil surface per unit weight of dry soil, $M\,M^{-1}$; and

Q_1 = rate of substance loss by adsorption, $M\,L^{-3}\,T^{-1}$.

A first-order transformation is assumed to result from the effects of physical, chemical, and biological transformations or reactions during the transfer of leachate substances in the soil-water system. This is represented as

$$Q_2 = P\theta C \tag{3.8}$$

where P is the transformation coefficient, T^{-1}; and Q_2 is the rate of substance loss by transformation, $M\,L^{-3}\,T^{-1}$.

By substituting Equations (3.4), (3.5), (3.7), and (3.8) in Equation (3.1), the governing transport equation for a one-dimensional leachate movement due to hydrodynamic dispersion, molecular diffusion, flow convection, linear equilibrium adsorption, and first-order transformation is

$$\frac{\partial(\theta C)}{\partial t} = \frac{\partial}{\partial x}\left[D\,\frac{\partial(\theta C)}{\partial x}\right] - \frac{\partial(V\theta C)}{\partial x} - \rho K \frac{\partial C}{\partial t} - P\theta C \tag{3.9}$$

If the volumetric moisture content θ is constant over space and time, the average pore-water velocity V is constant over space, and the apparent dispersion coefficient D is constant over space, then Equation (3.9) can be written as

$$(1 + \frac{\rho K}{\theta}) \frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2} - V \frac{\partial C}{\partial x} - PC \qquad (3.10)$$

or

$$\frac{\partial C}{\partial t} = \frac{1}{R} (D \frac{\partial^2 C}{\partial x^2} - V \frac{\partial C}{\partial x} - PC)$$

$$= D' \frac{\partial^2 C}{\partial x^2} - V' \frac{\partial C}{\partial x} - P'C \qquad (3.11)$$

where

$R = 1 + \rho K/\theta$, a unitless retardation factor;

$D' = D/R$, effective dispersion or dispersion-diffusion coefficient, $L^2\ T^{-1}$;

$V' = V/R$, effective average pore-water or seepage velocity, $L\,T^{-1}$; and

$P' = P/R$, effective transformation coefficient, T^{-1}.

The retardation factor, R, introduced by Hashimoto et al. (1964) is defined as the mean velocity of the moving solution relative to the mean velocity of its constituents as they move through the porous media. For linear equilibrium adsorption, the retardation factor is independent of the solute concentration. When no adsorption occurs, $K=0$, the retardation factor is equal to one. As the adsorption coefficient K is increased, so is the retardation factor; and the effective values for the dispersion-diffusion, the pore-water velocity, and the transformation coefficient are reduced. The net effect is to reduce the mobility of the solute transport.

In order to solve the governing transport Equation (3.11), one initial condition and the upper- and lower-boundary conditions are required.

This can be seen as integrating once over time and twice over space. For the initial condition, assume the background concentration is zero in the soil media originally. For the upper boundary, where leachate begins to flow through the liner of landfills, Raveh (1979) observed declining concentration of various substances in the leachate and described the concentration of various substances with an exponential function of time. The initial and upper-boundary conditions can be expressed as

$$C(x,0) = 0 \qquad \text{for } x > 0 \tag{3.12}$$

and

$$C(0,t) = C_o e^{-Gt} \qquad \text{for } t \geq 0 \tag{3.13a}$$

respectively, where $C_o = C(0,0)$, initial concentration at distance zero and time zero, ML^{-3}; and G is the decay coefficient for the source of substance, T^{-1}. For the lower boundary, the following condition is applied:

$$\frac{\partial C}{\partial x}(\infty,t) = 0 \tag{3.13b}$$

The problems of the lower boundary were extensively discussed by Danckwerts (1953), Wehner and Wilhelm (1956), Pearson (1959), Van Genuchten and Wierenga (1974), and Bear (1979). They all concluded that $\partial C/\partial x$ should be zero at the lower boundary to avoid a discontinuous concentration. A zero concentration at the lower boundary may be applicable to an infinite distance system; however, it is not adequate to apply this condition to a finite distance system.

Theoretically, leachate is not a single compound or substance, and the geological condition is not a single formation either. In order to use the above equations in a reasonable sense, the following assumptions are made:

1. Only hydrodynamic dispersion, molecular diffusion, flow convection, linear equilibrium adsorption, and first-order transformation processes are involved in the leachate transport.

2. Only a one-dimensional flow condition is considered and the lateral molecular diffusion is neglected.

3. Darcy's law is valid, and hydraulic-head gradients are the only significant driving force for the leachate flow.

4. Each soil layer is considered homogeneous and isotropic and has a constant moisture content and invariant coefficients of dispersion-diffusion, convection, linear adsorption, and first-order transformation.

5. The gradients of leachate density, concentration, viscosity, and temperature do not affect the transport parameters, coefficients, and processes.

6. No chemical reactions occur that change the leachate properties or the soil properties.

7. The release of leachate substances or constituents is continuous, and the background concentration in the soil is zero originally.

Numerical Model

A finite difference numerical model is developed to solve the governing transport Equation (3.11) based on the Crank-Nicolson method. In this method, the region of integration is covered by a finite difference mesh, and the finite difference solution is defined at the mesh intersections, called mesh or node points. The mesh spacings are assumed to be equally spaced. Although the method is based on the Crank-Nicolson method, there are three finite difference approximations, namely, centered-in-space, backward-in-space, and forward-in-space. The derivation of these difference equations is shown in Appendix A.

For the Crank-Nicolson method with the centered-in-space approximation, Equation (3.11) can be written in finite difference form as

$$\frac{C_i^{n+1} - C_i^n}{\Delta t} = D' \left(\frac{1}{2}\right)\left[\frac{C_{i+1}^{n+1} - 2C_i^{n+1} + C_{i-1}^{n+1}}{(\Delta x)^2} + \frac{C_{i+1}^n - 2C_i^n + C_{i-1}^n}{(\Delta x)^2}\right]$$
$$- V' \left(\frac{1}{2}\right)\left[\frac{C_{i+1}^{n+1} - C_{i-1}^{n+1}}{2\Delta x} + \frac{C_{i+1}^n - C_{i-1}^n}{2\Delta x}\right]$$
$$- P' \left(\frac{1}{2}\right)[C_i^{n+1} + C_i^n] \tag{3.14}$$

For the Crank-Nicolson method with the backward-in-space approximation, Equation (3.11) can be written as

$$\frac{C_i^{n+1} - C_i^n}{\Delta t} = D' \left(\frac{1}{2}\right)\left[\frac{C_{i+1}^{n+1} - 2C_i^{n+1} + C_{i-1}^{n+1}}{(\Delta x)^2} + \frac{C_{i+1}^n - 2C_i^n + C_{i-1}^n}{(\Delta x)^2}\right]$$
$$- V' \left(\frac{1}{2}\right)\left[\frac{C_i^{n+1} - C_{i-1}^{n+1}}{\Delta x} + \frac{C_i^n - C_{i-1}^n}{\Delta x}\right]$$
$$- P' \left(\frac{1}{2}\right)[C_i^{n+1} + C_i^n] \tag{3.15}$$

For the Crank-Nicolson method with the forward-in-space approximation, Equation (3.11) can be written as

$$\frac{C_i^{n+1} - C_i^n}{\Delta t} = D' \left(\frac{1}{2}\right)\left[\frac{C_{i+1}^{n+1} - 2C_i^{n+1} + C_{i-1}^{n+1}}{(\Delta x)^2} + \frac{C_{i+1}^n - 2C_i^n + C_{i-1}^n}{(\Delta x)^2}\right]$$
$$- V' \left(\frac{1}{2}\right)\left[\frac{C_{i+1}^{n+1} - C_i^{n+1}}{\Delta x} + \frac{C_{i+1}^n - C_i^n}{\Delta x}\right]$$

$$- P' \left(\frac{1}{2}\right)[C_i^{n+1} + C_i^n] \qquad (3.16)$$

where Δx, Δt are space and time increments, respectively; and i, n denote space position and time level, respectively.

Once rearranging the above three difference equations to a form such that:

$$A_1 C_{i-1}^{n+1} + A_2 C_i^{n+1} + A_3 C_{i+1}^{n+1} = B_1 C_{i-1}^n + B_2 C_i^n + B_3 C_{i+1}^n \qquad (3.17)$$

then, for the centered-in-space approximation in Equation (3.14):

$$A_1 = (-2D' - V'\Delta x)\Delta t$$

$$A_2 = 4(\Delta x)^2 + [4D' + 2P'(\Delta x)^2]\Delta t$$

$$A_3 = (-2D' + V'\Delta x)\Delta t$$

$$B_1 = -A_1$$

$$B_2 = -A_2 + 8(\Delta x)^2$$

$$B_3 = -A_3 \qquad (3.18)$$

For the backward-in-space approximation in Equation (3.15):

$$A_1 = (-D' - V'\Delta x)\Delta t$$

$$A_2 = 2(\Delta x)^2 + [2D' + V'\Delta x + P'(\Delta x)^2]\Delta t$$

$$A_3 = -D'\Delta t$$

$$B_1 = -A_1$$

$$B_2 = -A_2 + 4(\Delta x)^2$$

$$B_3 = -A_3 \qquad (3.19)$$

and for the forward-in-space approximation in Equation (3.16):

$$A_1 = -D'\Delta t$$

$$A_2 = 2(\Delta x)^2 + [2D' - V'\Delta x + P'(\Delta x)^2]\Delta t$$

$$A_3 = (-D' + V'\Delta x)\Delta t$$

$$B_1 = -A_1$$

$$B_2 = -A_2 + 4(\Delta x)^2$$

$$B_3 = -A_3 \qquad (3.20)$$

Equation (3.17) can now be written in matrix form, excluding the upper-boundary point, where $C_1^{n+1} = C_o e^{-G(n\Delta t)}$ (Equation (3.21), page 31).

By defining $C_{i+1}^{n+1} = C(i\Delta x, n\Delta t)$ and applying the upper-boundary condition, $C_1^{n+1} = C(0, n\Delta t) = C_o e^{-G(n\Delta t)}$, to Equation (3.21) the first element on the right-hand-side column matrix becomes

$$B_1C_1^n + B_2C_2^n + B_3C_3^n - A_1C_1^{n+1} = B_1C_oe^{-G(n-1)\Delta t}$$
$$+ B_2C_2^n + B_3C_3^n - A_1C_oe^{-G(n\Delta t)}$$

The last element on the right-hand-side column matrix is theoretically impossible to evaluate. Thus, assumptions have to be made before calculations can be carried on. In numerical methods, a finite distance is used instead of an infinite distance as used in analytical methods. Replacing the lower-boundary condition in Equation (3.13b) with

$$\frac{\partial C}{\partial x}(L,t) = 0 \qquad (3.22)$$

where L is the simulation distance for a soil layer. This assumption is applicable so long as L is reasonably larger than the distance to the

$$
\begin{bmatrix}
A_2 & A_3 & & & & & \\
A_1 & A_2 & A_3 & & \bigcirc & & \\
 & A_1 & A_2 & A_3 & & & \\
 & & \cdot & \cdot & \cdot & & \\
 & & & \cdot & \cdot & \cdot & \\
 & & & & \cdot & \cdot & \cdot \\
 & & \bigcirc & & A_1 & A_2 & A_3 \\
 & & & & & A_1 & A_2
\end{bmatrix}
\cdot
\begin{Bmatrix}
C_2^{n+1} \\ C_3^{n+1} \\ C_4^{n+1} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ C_{k-1}^{n+1} \\ C_k^{n+1}
\end{Bmatrix}
=
\begin{bmatrix}
B_1 C_1^n + B_2 C_2^n + B_3 C_3^n - A_1 C_1^{n+1} \\
B_1 C_2^n + B_2 C_3^n + B_3 C_4^n \\
B_1 C_3^n + B_2 C_4^n + B_3 C_5^n \\
\cdot \\ \cdot \\ \cdot \\ \cdot \\
B_1 C_{k-2}^n + B_2 C_{k-1}^n + B_3 C_k^n \\
B_1 C_{k-1}^n + B_2 C_k^n + B_3 C_{k+1}^n - A_3 C_{k+1}^{n+1}
\end{bmatrix}
\tag{3.21}
$$

end of that soil layer. In this case, C_{k+1}^{n} and C_{k+1}^{n+1} are both unknowns and cannot be evaluated. Assuming C_{k+1}^{n} and C_{k+1}^{n+1} are equal to C_{k}^{n}, the last element on the right-hand-side column matrix becomes

$$B_1 C_{k-1}^{n} + B_2 C_{k}^{n} + B_3 C_{k+1}^{n} - A_3 C_{k+1}^{n+1}$$

$$= B_1 C_{k-1}^{n} + (B_2 + B_3 - A_3)\ C_{k}^{n}$$

The calculation starts with initial and boundary conditions, i.e., $n = 1$, and

$$
\begin{aligned}
C_1^1 &= C(0,0) = C_o \\
C_1^2 &= C(0,\Delta t) = C_o e^{-G\Delta t} \\
C_{i+1}^1 &= C(i\Delta x,0) = 0 \qquad \text{for } 1 \le i \le k \\
C_{k+1}^2 &= C(L+\Delta x,\Delta t) = 0
\end{aligned}
\tag{3.23}
$$

For each time step, the computer solves a system of equations covered from the beginning of a soil layer to the simulation distance for that particular soil layer. The calculation continues one time step by one time step until the simulation period is reached. The technique used to solve the system of difference equations is based on the Gauss elimination method for tridiagonal matrices. Tridiagonal matrices are special cases of banded matrices which contain relatively few nonzero elements about the main diagonal. For a tridiagonal matrix, there are only three nonzero elements, as A_1, A_2, and A_3 in Equation (3.21), on the diagonals.

The Gauss elimination method is very simple and efficient for tridiagonal systems. The whole operation actually only alters the main diagonal elements and the right-hand-side elements. At the first stage, only the second element on the first column needs to be eliminated and this elimination only affects the second elements on the second and the right-hand-side columns. The first reduced system of equations is again tridiagonal and so are the subsequent reduction stages. Each forward reduction step requires only one division, two multiplications, and two subtractions. There are (N - 1) stages for a tridiagonal system of N equations. Hence, the forward reduction requires (N - 1) divisions, 2(N - 1) multiplications, and 2(N - 1) subtractions to reduce the system to an upper triangular matrix.

The backward substitution is also simple. Each equation has two unknowns except the last equation which now has only one unknown. Hence, the backward substitution requires N divisions, (N - 1) multiplications, and (N - 1) subtractions to find the solutions. Therefore, the total operations required for a tridiagonal system of equations using the Gauss elimination method are (2N - 1) divisions, 3(N - 1) multiplications, and 3(N - 1) subtractions. For a full matrix, this requires a function of N^3 operations (Ortega and Poole, 1981). Hence, the Gauss elimination method is very efficient for a tridiagonal system, since the operations required only increase linearly with the number of equations N. Also, the computer storage needed for a banded tridiagonal matrix is minimal. In this study, only the main diagonal elements, unknown vector, and right-hand-side vector are stored. The storage requirement for solving a tridiagonal system of equations is 3N locations, compared with $(N^2 + 2N)$ locations for a full matrix.

The same solution methods are also applied to the multilayered soil media. The only difference is to replace the boundary condition at the interface between two soil layers as the upper-boundary condition for the lower layer. This is necessary in order to maintain the solute concentration continuity at the interface between any two soil layers. The lower-boundary condition is the same for every layer, except we may use different simulation distances. The assumption is that the concentrations in the upper layers are not affected by the presence of the lower layers. This implies that the changes in any transport coefficients are not affected by the upper layers. The experimental results and numerical solutions presented by Shamir and Harleman (1967) for a simple, dispersion and convection only, transport equation indicate that the above assumption is appropriate.

Analytical Model

An analytical model is not available for the leachate transport in multilayered soil media. However, in order to test the validity of the numerical model developed, an analytical model is adopted from Van Genuchten and Alves (1982) to compare with the numerical model for one-dimensional leachate transport in a single-layered soil medium. The equation to represent the analytical model for Equations (3.11), (3.12), and (3.13) is

$$C(x,t) = \frac{1}{2} C_o e^{-Gt} \left\{ \exp\left[\frac{(V' - H)x}{2D'}\right] \operatorname{erfc}\left[\frac{x - Ht}{2(D't)^{\frac{1}{2}}}\right] + \exp\left[\frac{(V' + H)x}{2D'}\right] \operatorname{erfc}\left[\frac{x + Ht}{2(D't)^{\frac{1}{2}}}\right] \right\}$$

$$H = [V'^2 + 4D'(P' - G)]^{\frac{1}{2}} \qquad (3.24)$$

in which exp is the exponential function, and erfc is the complementary error function.

Let

$$R_1 = \frac{(V' - H)x}{2D'} \qquad R_2 = \frac{(V' + H)x}{2D'}$$

and

$$S_1 = \frac{x - Ht}{2(D't)^{\frac{1}{2}}} \qquad S_2 = \frac{x + Ht}{2(D't)^{\frac{1}{2}}}$$

Then

$$C(x,t) = \frac{1}{2} C_o e^{-Gt} \{\exp(R_1) \ \mathrm{erfc}(S_1) + \exp(R_2) \ \mathrm{erfc}(S_2)\} \tag{3.25}$$

To evaluate the product of the exponential function (exp) and the complementary error function (erfc), Van Genuchten and Alves (1982) defined a function EXF(A,B) as

$$\mathrm{EXF}(A,B) = \exp(A) \ \mathrm{erfc}(B) \tag{3.26}$$

where

$$\begin{aligned} \mathrm{erfc}(B) &= 1 - \mathrm{erf}(B) \\ &= 1 - \frac{2}{\pi^{\frac{1}{2}}} \int_o^B \exp(-u^2) \ du \\ &= \frac{2}{\pi^{\frac{1}{2}}} \int_B^\infty \exp(-u^2) \ du \end{aligned}$$

For $0 \leq B \leq 3$,

$$\begin{aligned} \mathrm{EXF}(A,B) &\simeq \exp(A - B^2)(u(a_1 + u(a_2 + u(a_3 + u(a_4 + ua_5))))) \\ &= \exp(A - B^2)(a_1 u + a_2 u^2 + a_3 u^3 + a_4 u^4 + a_5 u^5) \end{aligned} \tag{3.27}$$

where

$$u = \frac{1}{1 + 0.3275911\ B}$$

$$a_1 = 0.2548296$$

$$a_2 = -0.2844967$$

$$a_3 = 1.4214140$$

$$a_4 = -1.4531520$$

$$a_5 = 1.0614050$$

For B > 3,

$$\mathrm{EXF}(A,B) \simeq \frac{1}{\pi^{\frac{1}{2}}} \exp(A - B^2)/(B + 0.5/(B + 1.0/(B + 1.5/(B + 2.0/(B + 2.5/(B + 1.0)))))) \qquad (3.28)$$

When B < 0,

$$\mathrm{EXF}(A,B) = 2\exp(A) - \mathrm{EXF}(A,-B) \qquad (3.29)$$

The last term in Equation (3.29) is evaluated from Equation (3.27) or (3.28), depending on the value of -B. For large and small values of A and B, the function EXF(A,B) cannot be used and a zero is assigned for EXF(A,B), such as with the following two conditions:

$$\text{If } |A| > 170 \quad \text{and} \quad B \le 0,\ \mathrm{EXF}(A,B) = 0$$

$$\text{If } |A - B^2| > 170 \quad \text{and} \quad B > 0,\ \mathrm{EXF}(A,B) = 0$$

With the above method, the computer can produce an accuracy of at least four digits for the analytical solution of Equations (3.11), (3.12), and (3.13).

CHAPTER IV

MODEL VERIFICATION

One of the tasks which must be carried out in obtaining a numerical solution to any problem is to verify that the computer program and the final solution are correct. Verification often is carried out by comparing the model with an available analytical model and/or a numerical model. Since the correct solution to a problem is usually unknown, the verification procedure will usually be indirect. This indirect approach consists of various limiting cases of the problem for which known solutions are available. These limiting cases could be simulated with the program under consideration by setting certain terms to zero, letting certain parameters or coefficients become very small, or bypassing certain sections of the program temporarily. In this study, no comparable numerical model was found and the existing analytical model is only available for transports in single-layered soil media. An analytical model developed by Van Genuchten and Alves (1982) is thus adopted to compare with the numerical model for a one-dimensional leachate transport.

Model Description

The computer program is written in WATFIV and run on an IBM-3081K computer. The program consists of the main program, six subroutines, and one function. The main program reads input data, prints output data, and calls three or four subroutines, depending on the number of soil layers. If the number of layers is one, it will call four subroutines--ANALY, CNCENT, CNBACK, and CNFORW--to compute one analytical and three numerical solutions, respectively. If the number of layers is greater than one, it will only call three subroutines--CNCENT, CNBACK, and CNFORW--to compute three numerical solutions.

The first subroutine ANALY coordinates with a subprogram function EXF(A,B) and computes the analytical solution for a single soil layer. However, if the effective dispersion coefficient D' is less than or equal to zero, or $H1 = V'^2 + 4D'(P - G)$ is negative in the square root (DSQRT), the computer will bypass the analytical solution and compute only numerical solutions.

Subroutines CNCENT, CNBACK, and CNFORW calculate the left-hand-side tridiagonal elements and the right-hand-side coefficients based on the Crank-Nicolson method with the centered-in-space, backward-in-space, and forward-in-space approximations, respectively. Each of them calls subroutine SOLVE to compute the right-hand-side vector and then use the subroutine GAUSS to solve the tridiagonal system of equations. The computed values are then used to reestablish the right-hand-side vector. The procedure is repeated until the end of the simulation time. The solution technique used in subroutine GAUSS is based on the Gauss elimination method for tridiagonal matrices.

The computer program is listed in Appendix B, and the input data sequence and format type are presented in Appendix C. The input data are written in free format.

Model Verification

The model is verified by comparing the numerical solutions with the analytical solution in a one-dimensional transport through single-layered soil media. Different space and time increments, Δx and Δt, are used to test the accuracy.

The results are shown in Figures 1 through 4, assuming the effective dispersion coefficient D' is 1.0 m^2/year, the effective average pore-water velocity V' is 1.0 m/year, the effective transformation coefficient P' is 0.1 $year^{-1}$, and the decay coefficient for the source of substance G is 0.05 $year^{-1}$. Note that the numerical model agrees with the analytical model very well and gives almost the exact solution when space and time increments are 0.1 meter and 0.1 year, respectively. Even with a large increment of time, the results are still very satisfactory. This can be seen in Figure 2, where the Δt is 1.0 year. As the increment of space increases, the deviation between the numerical and analytical solutions increases for both backward-in-space and forward-in-space approximations. This can be seen in Figures 3 and 4, where both have the same Δx, 1.0 meter, but different Δt's, 1.0 and 0.1 year, respectively. The difference between the numerical and analytical solutions varies with time and distance from the source. The results show that the centered-in-space approximation has the best accuracy among the three approximations. However, with small increments in both space and time, three approximations will provide solutions close to the analytical solution. It is interesting to note that the solution from the centered-in-space approximation is about the average of the solutions from the backward-in-space and forward-in-space approximations.

For multilayered soil media, the analytical solution is not available and alternative means must be used to test the validity of the model developed. One of the two methods used in this study is to divide a

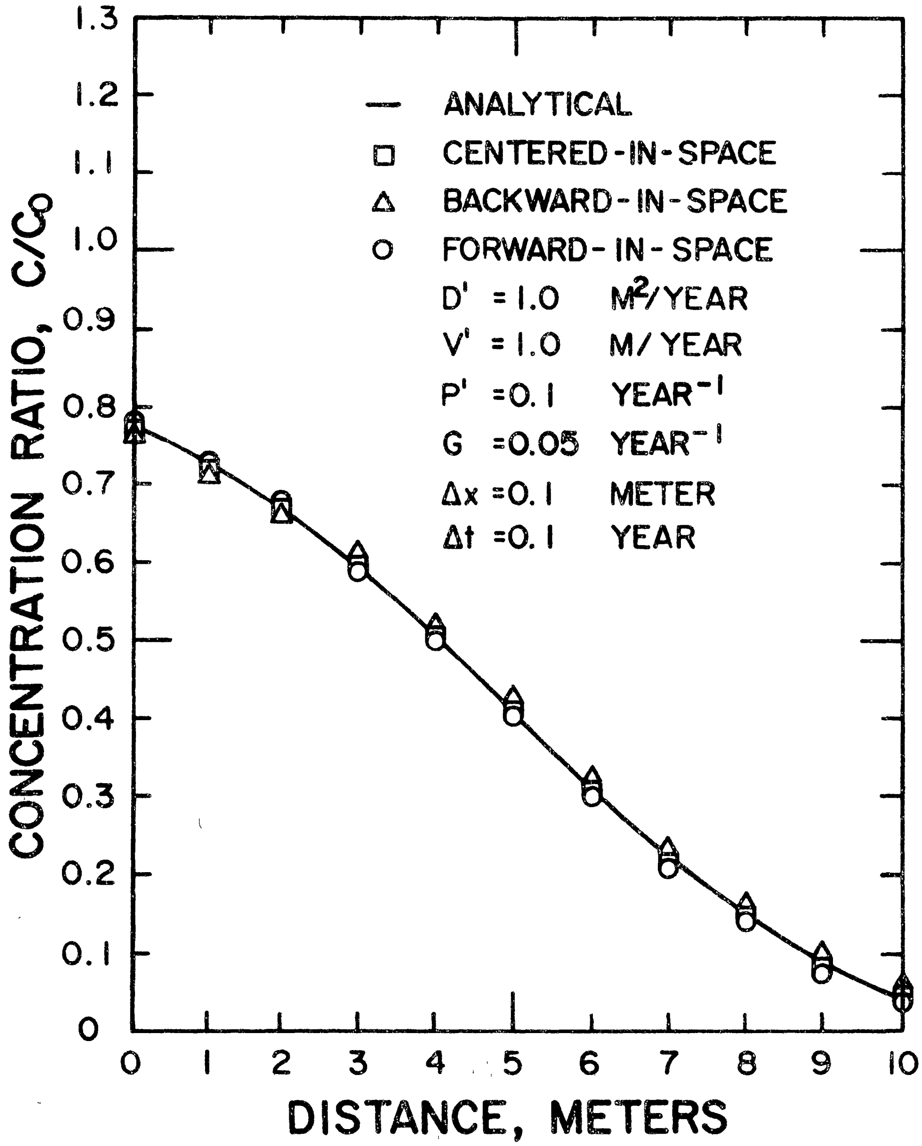

Figure 1. Comparison of Numerical and Analytical Solutions With Δx = 0.1 Meter, Δt = 0.1 Year, and Time of Simulation 5 Years

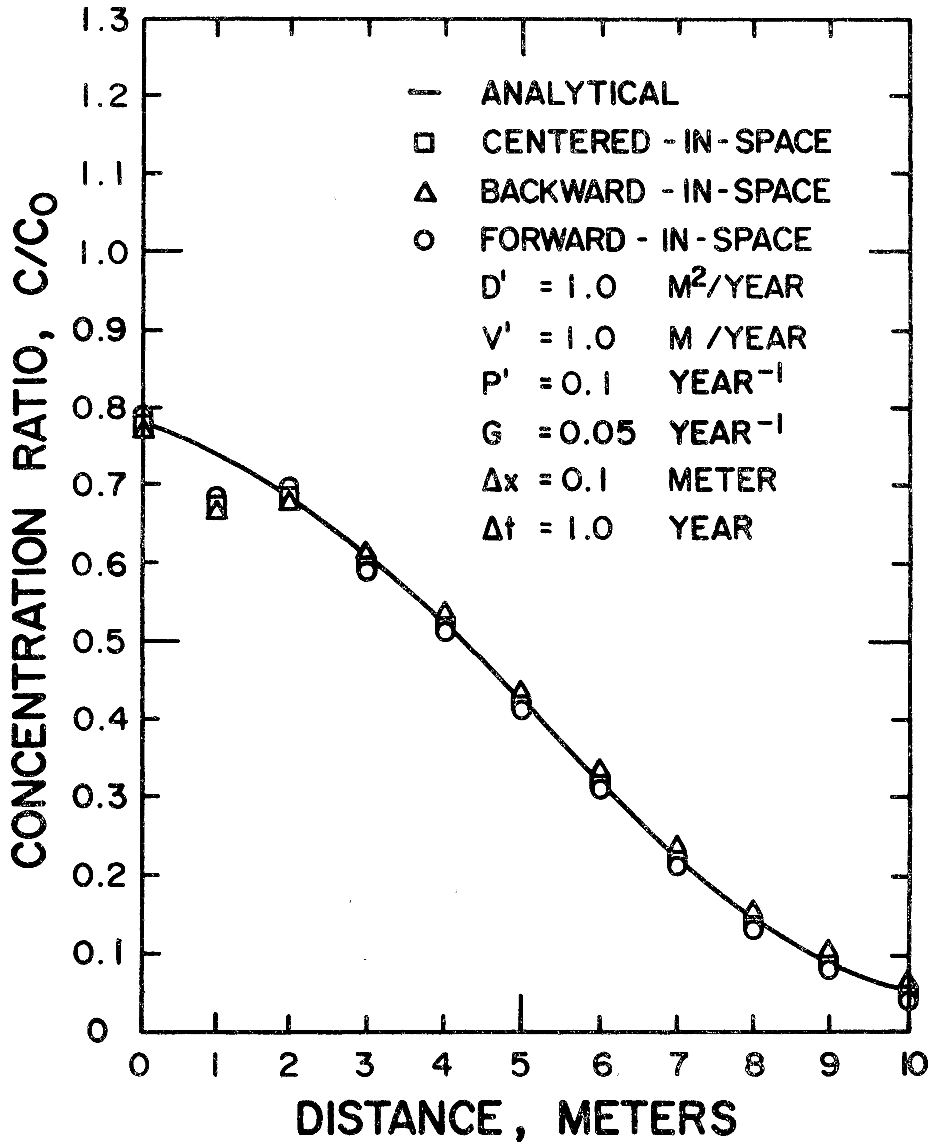

Figure 2. Comparison of Numerical and Analytical Solutions With $\Delta x = 0.1$ Meter, $\Delta t = 1.0$ Year, and Time of Simulation 5 Years

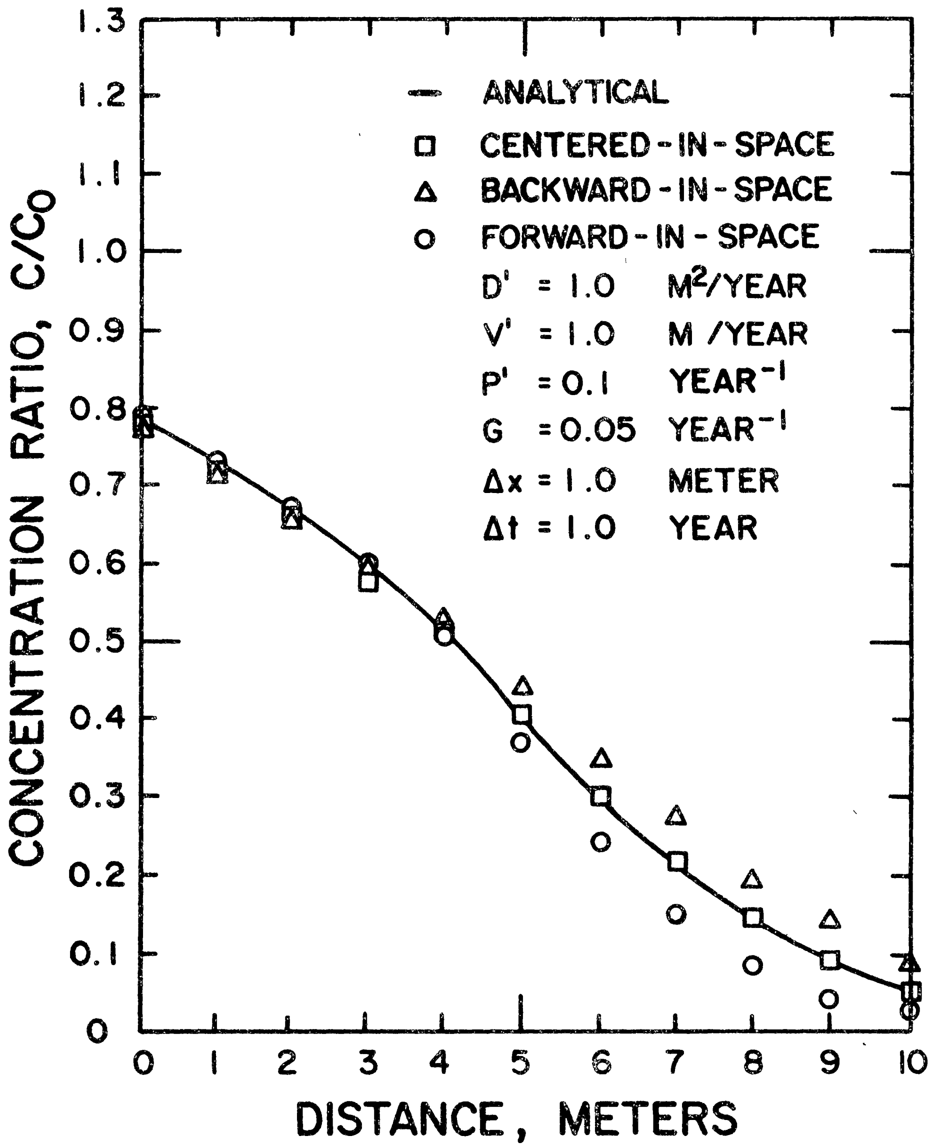

Figure 3. Comparison of Numerical and Analytical Solutions With Δx = 1.0 Meter, Δt = 1.0 Year, and Time of Simulation 5 Years

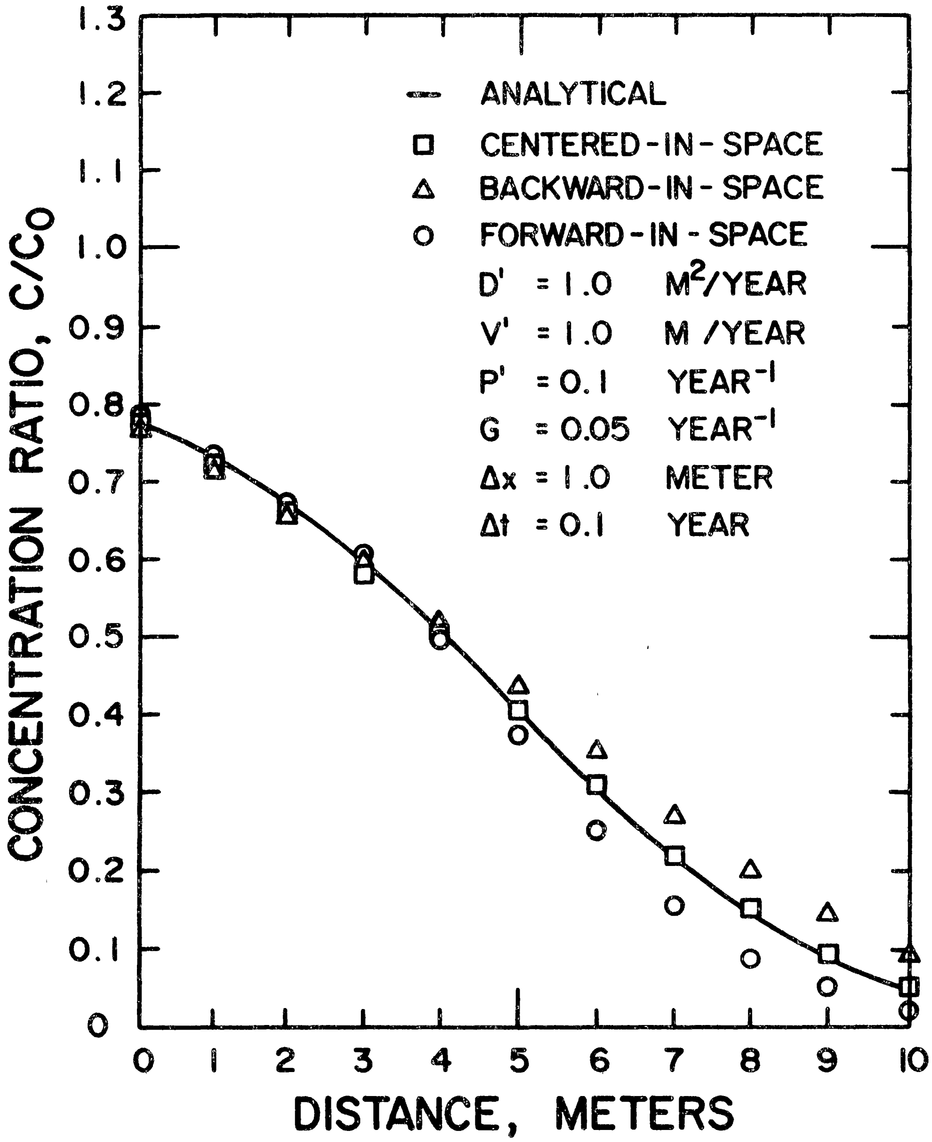

Figure 4. Comparison of Numerical and Analytical Solutions With Δx = 1.0 Meter, Δt = 0.1 Year, and Time of Simulation 5 Years

homogeneous and isotropic soil matrix into two, three, or more sections. Each section is considered as a layer, with all layers having the same transport parameters and coefficients. The results are then compared with the analytical and numerical solutions for a single layer with a thickness equaling that of all sections combined. The results of testing are identical to the one shown in Figure 1. The other method is to change the order of soil layers, with each layer possessing different transport parameters and coefficients, and to compare the effluent concentrations. Two different arrangements for a five-layered soil medium have been conducted and the results are shown in Figures 5 and 6. The output data are listed in Appendix D (refer to Test Data No. 3 and No. 4). Figure 5 shows that the concentration distributions are different but the effluent concentrations are the same. The results in Figure 6 represent that the effluent concentrations at a distance of 10 meters are identical at any time. This agrees with the works by Shamir and Harleman (1967) and by Selim et al. (1977), which state that the order of layers does not influence the effluent concentration distribution.

Difficulties of Numerical Approximations

For certain types of problems or conditions, numerical difficulties may be encountered. Sometimes these difficulties reveal themselves as unrealistic or inaccurate results. Several of these numerical difficulties in transport problems are the numerical dispersion (or numerical smearing), overshoot, and oscillation caused by the sharp concentration front. A sharp concentration front refers to a large change in the concentration over a short distance. When the concentration front lacks dispersion, such as in a convection only (often referred to as piston

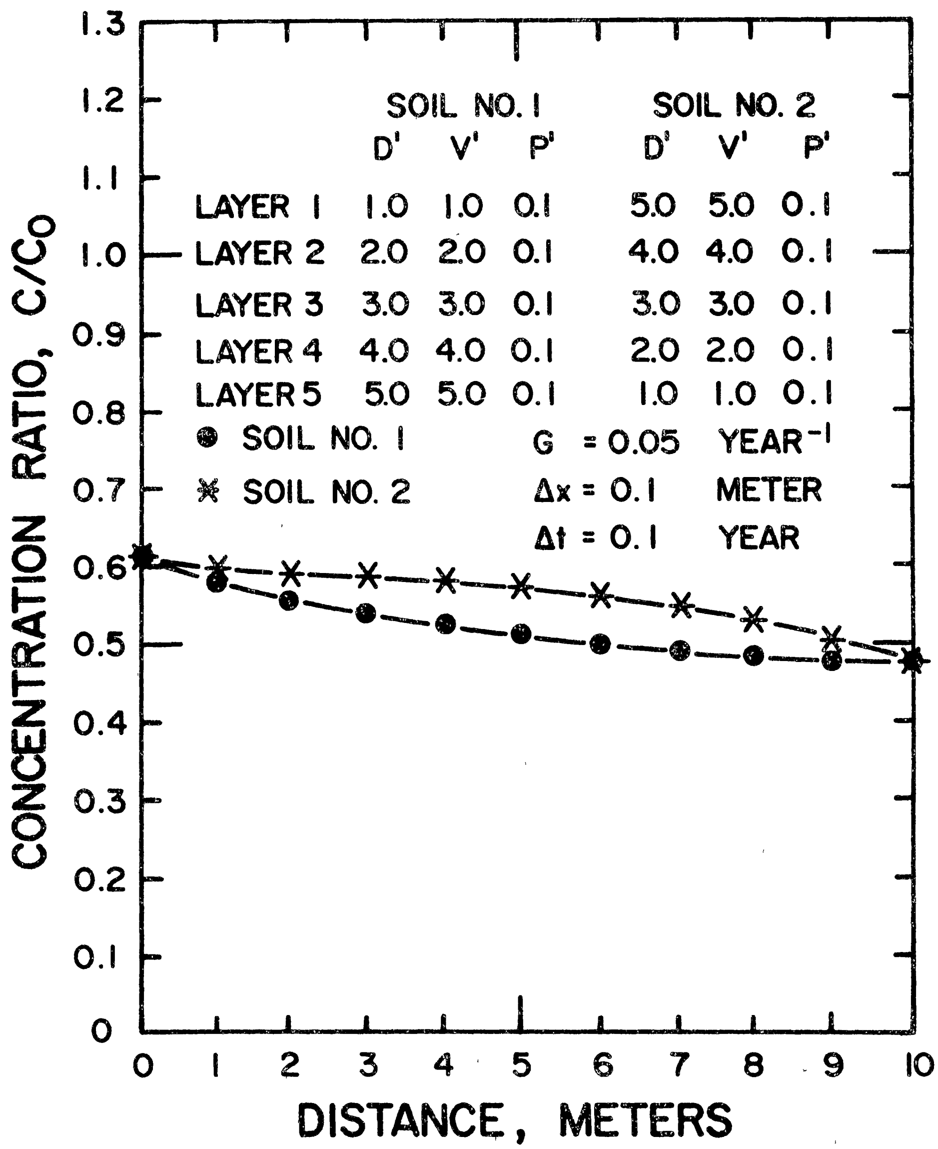

Figure 5. Spatial Concentration Distributions for Two Five-Layered Soil Media With Time of Simulation 10 Years and Thickness of Each Layer 2 Meters

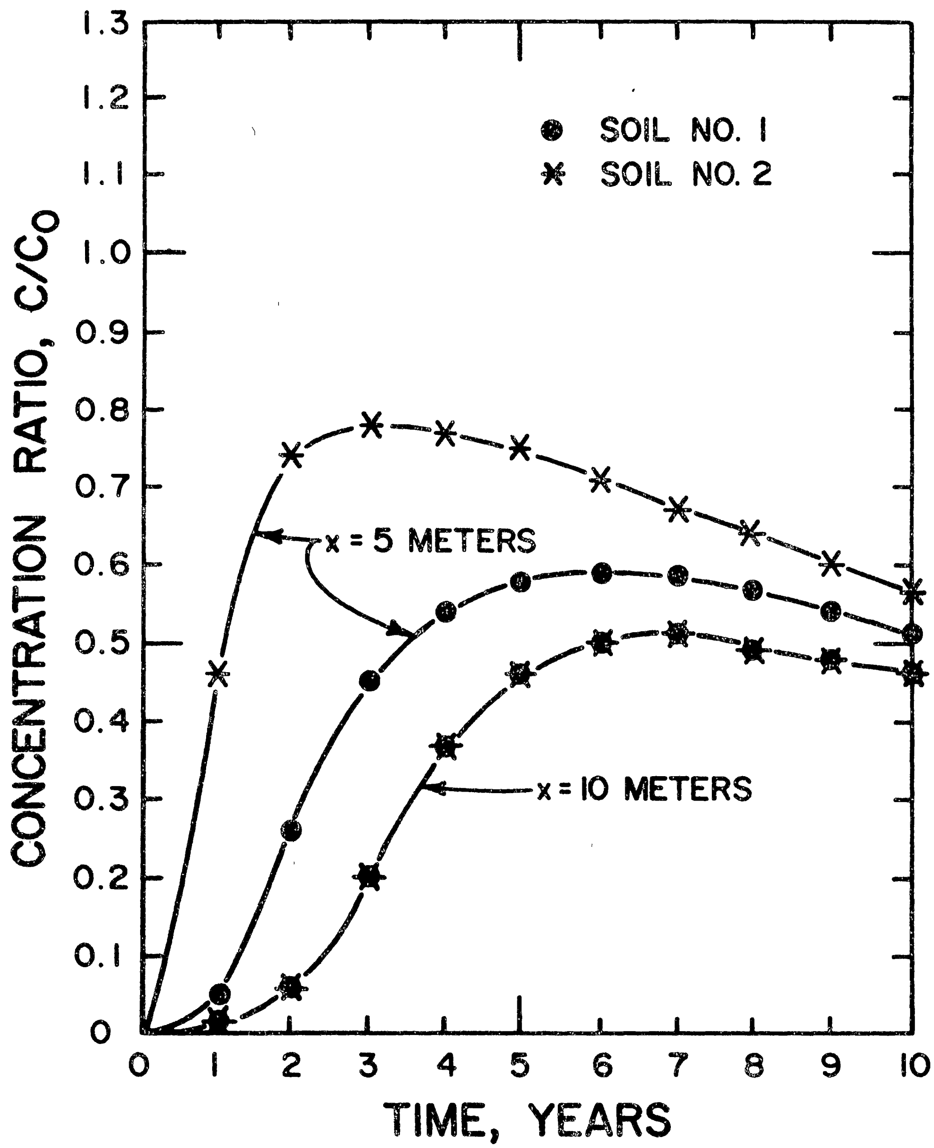

Figure 6. Temporal Concentration Distributions for Two Five-Layered Soil Media at Distances 5 and 10 Meters From the Source (Data Used are the Same as in Figure 5)

flow), it would appear as a sharp front that moves out at the average fluid velocity. But with dispersion, the front is "smeared out." The smearing of the concentration front due to numerical approximations to the transport equation is generally referred to as numerical dispersion or artificial dispersion, which acts exactly like the physical dispersion. While the numerical dispersion yields solutions that are smeared out, the oscillation results in solutions that are numerically unstable. The overshoot can often be explained as part of the oscillation; it generates solutions that are greater than the source concentration.

The numerical dispersion, overshoot, and oscillation are all related to the Peclet number (Pe) and the space and time step sizes. The Peclet number is defined as velocity times distance, divided by the dispersion-diffusion coefficient. While dispersion-diffusion is a measure of the "spreading" of the concentration front, the Peclet number is a measure of the significance of dispersion-diffusion, provided the distance is constant. When dispersion-diffusion does not occur, the Peclet number is infinitive.

Numerical dispersion is a truncation error. The one-sided backward and forward difference approximations to the first-order derivative in space generate an error term proportional to the second-order space derivative. This generated error term is called numerical dispersion or artificial dispersion, since it has the same effect as physical dispersion. The convection term approximated by the backward difference from Equation (A.19) is written as

$$-V' \left[\frac{C_i^{n+1} - C_{i-1}^{n+1} + C_i^n - C_{i-1}^n}{2\Delta x} \right] =$$

$$-V'\left\{\left[\frac{\partial C_i^{n+\frac{1}{2}}}{\partial x} - \frac{1}{2}\Delta x \frac{\partial^2 C_i^{n+\frac{1}{2}}}{\partial x^2} \pm \ldots\right]\right.$$

$$+ \frac{1}{8}\Delta t^2 \frac{\partial^2}{\partial t^2}\left[\frac{\partial C_i^{n+\frac{1}{2}}}{\partial x} - \frac{1}{2}\Delta x \frac{\partial^2 C_i^{n+\frac{1}{2}}}{\partial x^2}\right.$$

$$\left.\left.\pm \ldots\right] + \ldots\right\} \tag{4.1}$$

If the effective transformation coefficient P' = 0 and omitting the higher order terms, then

$$-V'\left[\frac{C_i^{n+1} - C_{i-1}^{n+1} + C_i^n - C_{i-1}^n}{2\Delta x}\right] \simeq -V' \frac{\partial C_i^{n+\frac{1}{2}}}{\partial x}$$

$$+ \frac{1}{2}\Delta x\, V' \frac{\partial^2 C_i^{n+\frac{1}{2}}}{\partial x^2} \tag{4.2}$$

A numerical dispersion coefficient $1/2\ \Delta x V'$ is thus introduced, which may be of the order of the physical dispersion coefficient D'. If $D' = \alpha V'$ and α is a constant, then the dispersion coefficient is $(\alpha + 1/2\ \Delta x)V'$ in the numerical computation. The numerical dispersion can be reduced if $\alpha V'$ is replaced by $(\alpha - 1/2\ \Delta x)V'$. Similarly, for the forward difference approximation from Equation (A.20), D' becomes $(\alpha - 1/2\ \Delta x)V'$, and the numerical dispersion can be corrected by replacing $\alpha V'$ with $(\alpha + 1/2\ \Delta x)V'$. For the central difference approximation from Equation (A.18) the highest truncation error is in terms of the third-order space derivative, which possesses the property of numerical error but differs from numerical dispersion.

If the effective transformation coefficient P' is also included, then the finite difference approximations will generate not only numerical dispersion but numerical convection and numerical transformation as well. The error analysis will become complicated and tedious. Thus, when the transport parameters are large, the units of the parameters might need to change in order to reduce the truncation errors. For very small transport parameters, the units may also need to change in order to avoid the computational difficulties. There are no specific criteria for how large or how small the parameters and step-sizes should be restricted. The general rules are to look into the relationships among the parameters, the Peclet number, the simulation distance and time, and the number of space and time steps, and to consider the requirements of the accuracy and computational cost. Usually, the centered- and forward-in-space approximations are applicable only for transports with lower Peclet numbers. However, a use of D', P', and V' between 1.0 and 0.01, and Δx and Δt no greater than 1.0 generally can provide a good numerical solution for a limited simulation distance.

The accuracy of the backward- and forward-in-space approximations is more dependent on Δx than on Δt, since both approximations are only first-order accurate in space but second-order accurate in time. The centered-in-space approximation, however, is second-order accurate in both space and time. Thus, the deviations shown in Figures 3 and 4 were mainly contributed to the numerical dispersion and other truncation errors in the higher-order time derivatives. The deviations arose because a larger space increment was used.

Figure 7 shows the results for a transport problem with D' = 0.01 m^2/year, V' = 1.0 m/year, P' = 0.0 $year^{-1}$, Δx = 0.01 meter, and Δt = 0.01 year. The simulation time is 0.5 year and the source concentration is constant. The distance of simulation is 1.1 meter and the Peclet number

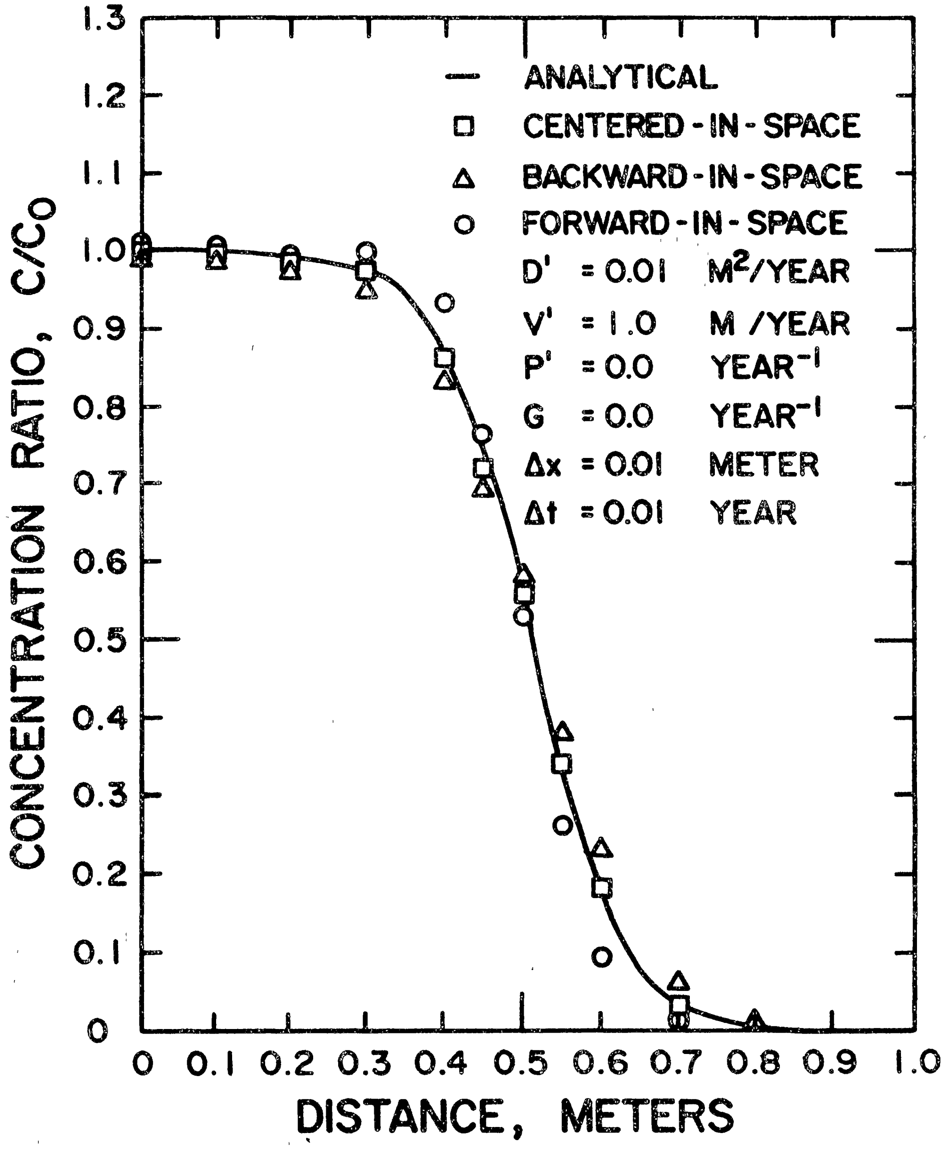

Figure 7. Comparison of Numerical and Analytical Solutions With Time of Simulation 0.5 Year and Peclet Number Pe = 110

is thus 110. Note that the dispersion is larger for the backward-in-space approximation and smaller for the forward-in-space approximation.

Numerical difficulties from the "exponent-exceeds-underflow limit" inside the computer may occur for the centered- and forward-in-space approximations, depending on whether the tridiagonal coefficient matrices are diagonally dominant or not. This relies on the values of D', V', P', Δx, and Δt in Equations (3.18) and (3.19). The simulation distance is also a factor to be considered. If the distance is too long, the soil media may not be able to "absorb" numerically the injected mass of substance. The distance and space and time steps should be chosen carefully and adapted to the problem. Since the transformation coefficient P' usually is relatively small, the Peclet number and step sizes are considered to be the most important factors for numerical stability and accuracy.

When there is no dispersion, the concentration front moves straightforward with time if the transformation is omitted. However, the numerical methods cannot obtain such a result in the rapid change of the concentration front. Some results are shown in Figures 8 through 12, using different space and time increments, where V' = 1.0 m/year, and D', P', and G are zero. The analytical model and the numerical model with the forward-in-space approximation cannot be applied since D' is zero. Note that the centered-in-space approximation tends to produce overshoot and oscillation, while the backward-in-space approximation tends to generate more numerical dispersion. It is interesting to note that for the centered-in-space approximation, the more space steps are used the more oscillations occur. However, the backward-in-space approximation is always stable, except when Δx and Δt are not compatible and Δt is signifi-

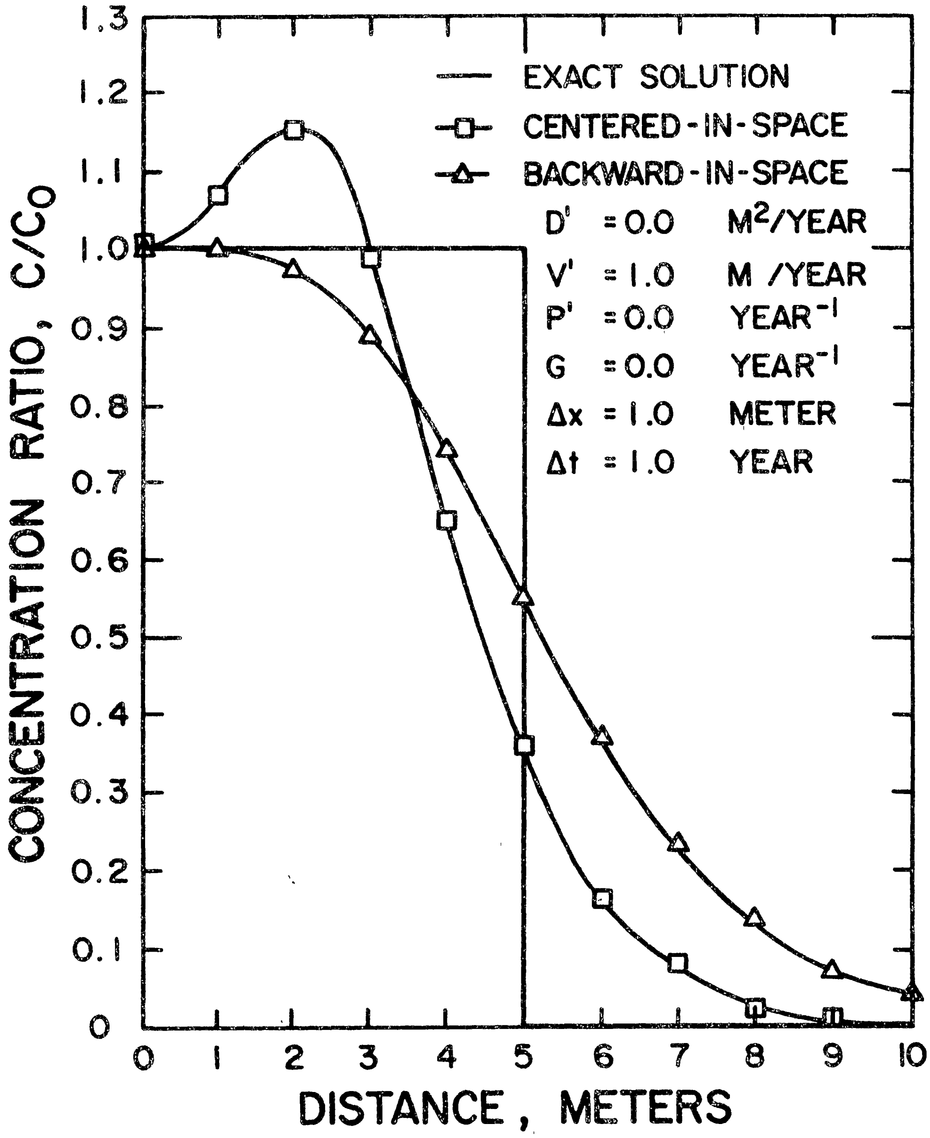

Figure 8. Numerical Dispersion and Overshoot in Convective Transport With $\Delta x = 1.0$ Meter, $\Delta t = 1.0$ Year, and Time of Simulation 5 Years

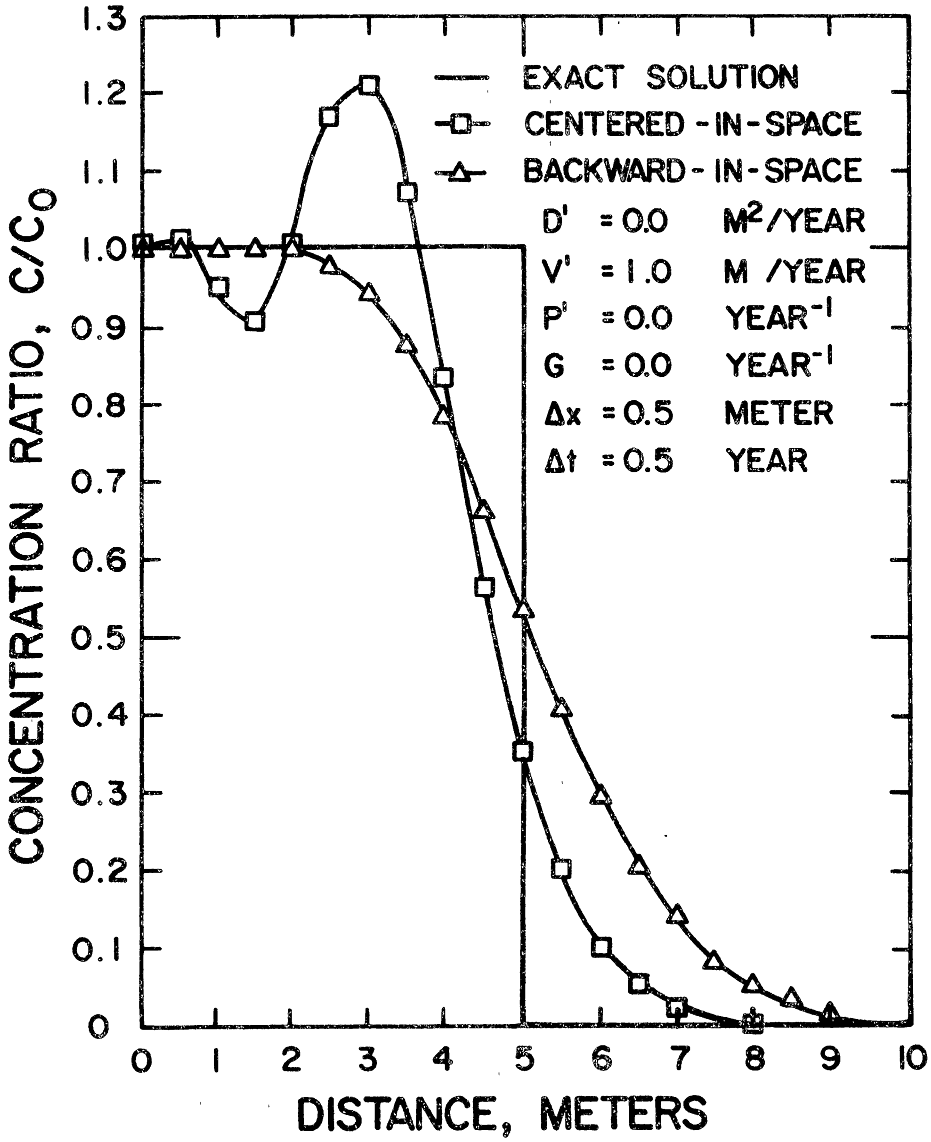

Figure 9. Numerical Dispersion and Oscillation in Convective Transport with Δx = 0.5 Meter, Δt = 0.5 Year, and Time of Simulation 5 Years

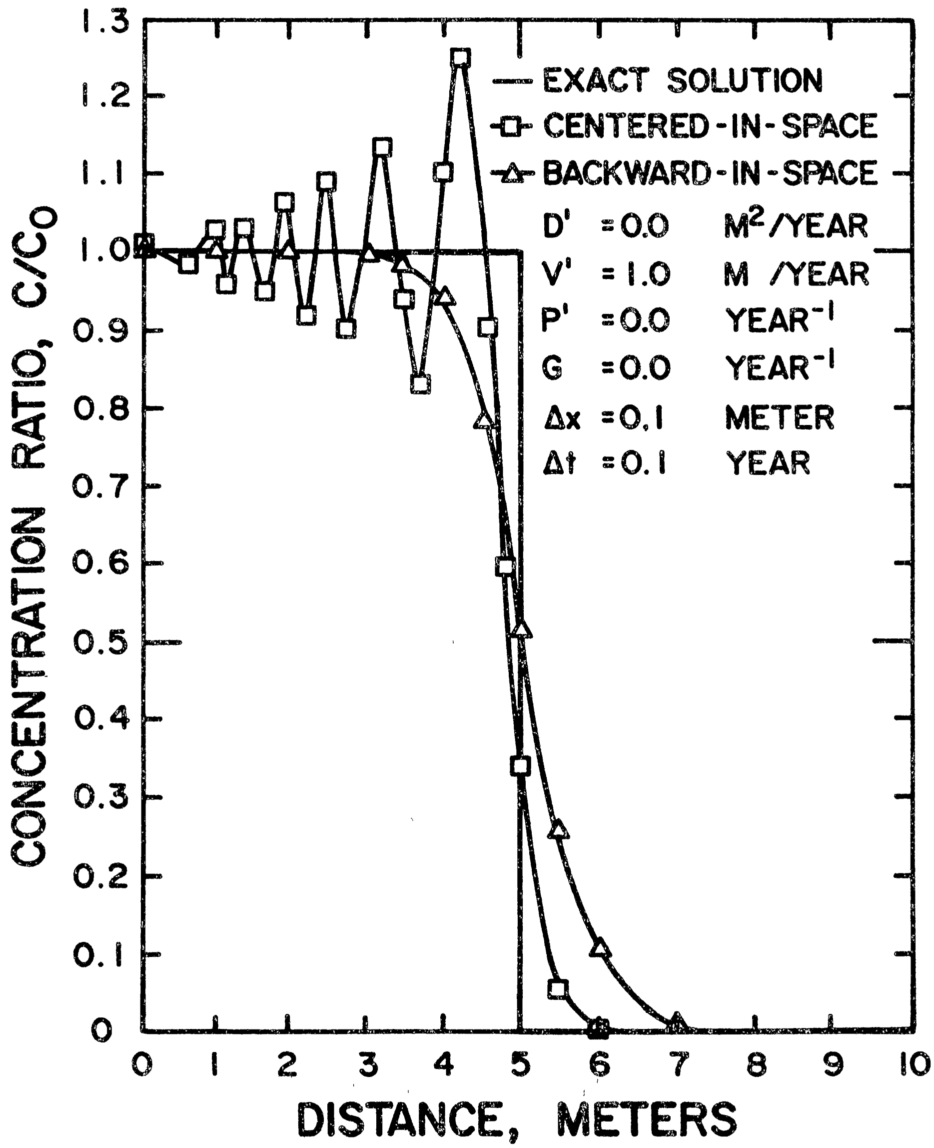

Figure 10. Numerical Dispersion and Oscillation in Convective Transport With Δx = 0.1 Meter, Δt = 0.1 Year, and Time of Simulation 5 Years

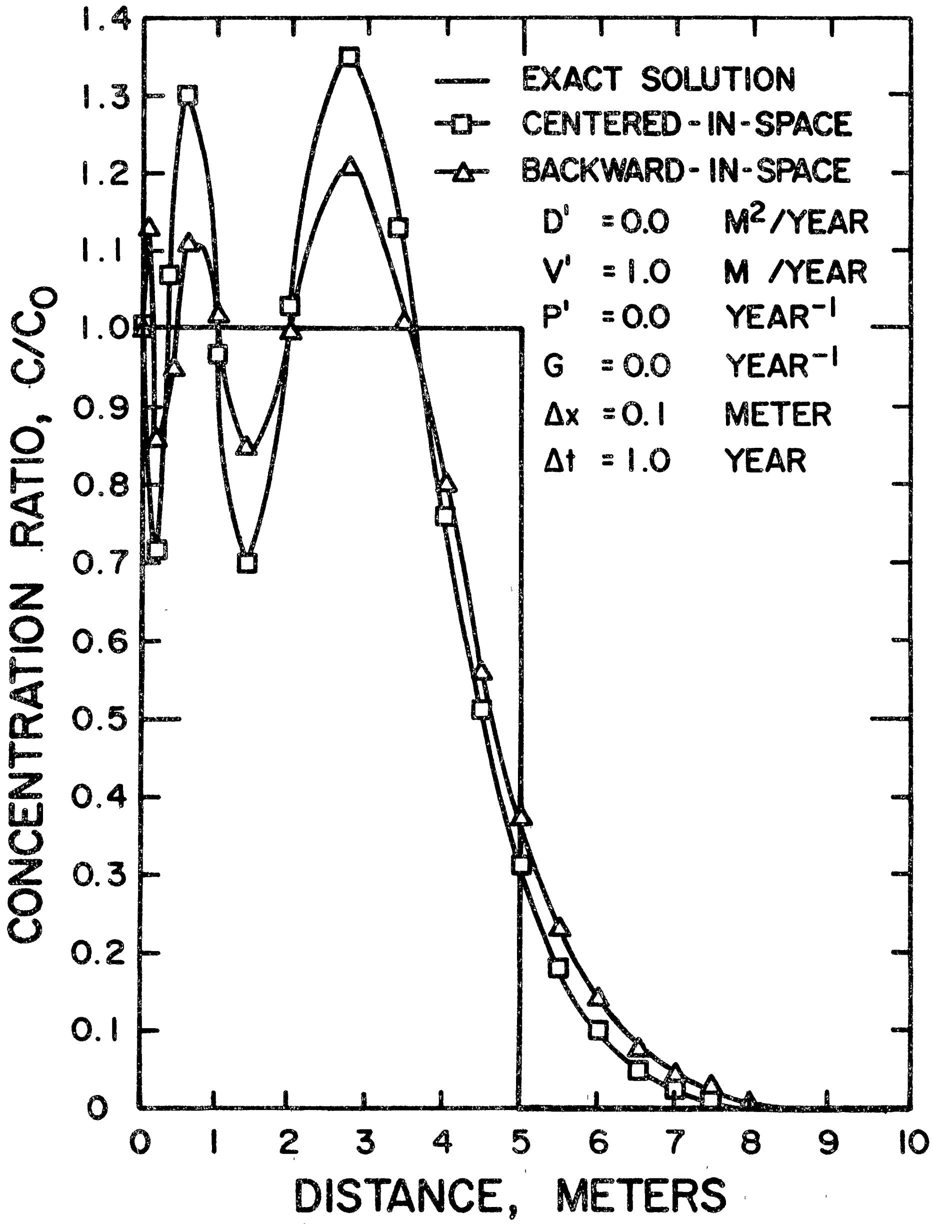

Figure 11. Numerical Dispersion and Oscillation in Convective Transport With Δx = 0.1 Meter, Δt = 1.0 Year, and Time of Simulation 5 Years

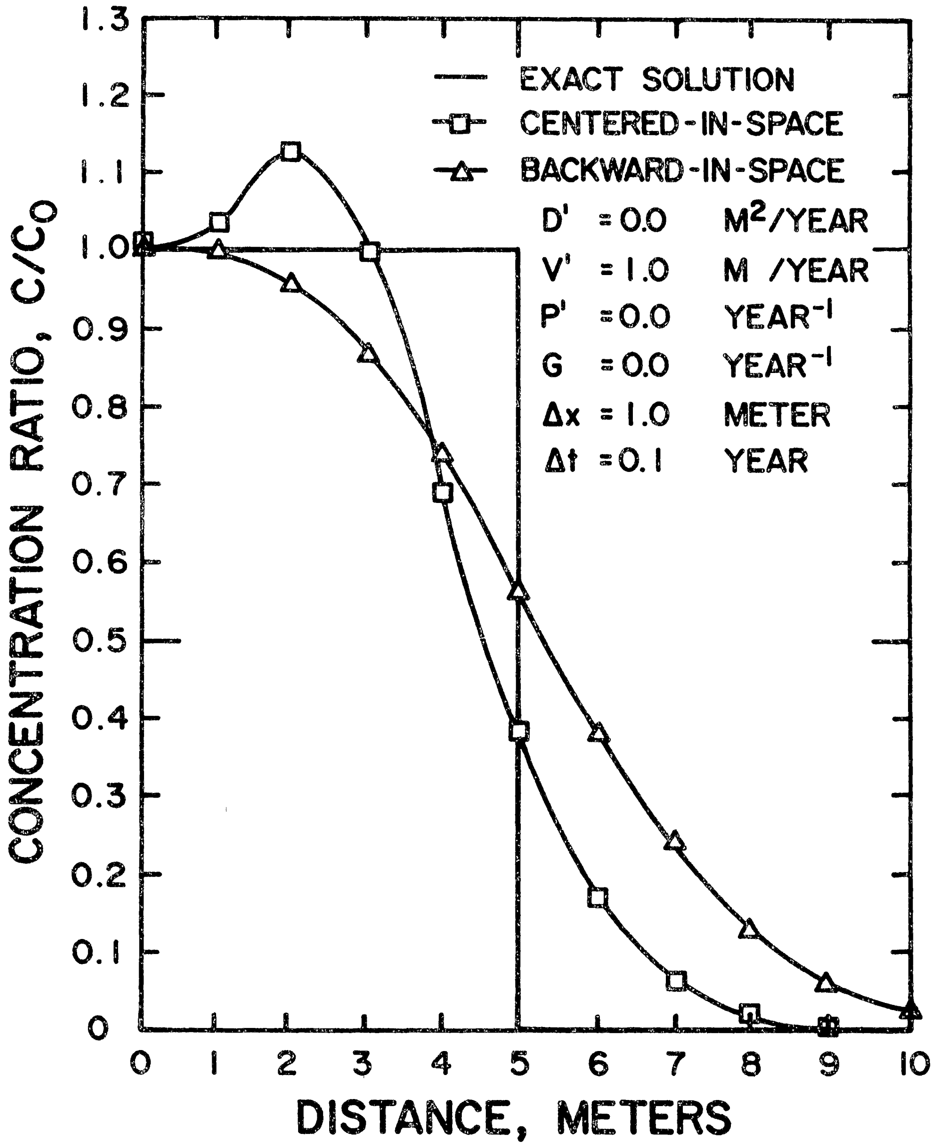

Figure 12. Numerical Dispersion and Overshoot in Convective Transport With $\Delta x = 1.0$ Meter, $\Delta t = 0.1$ Year, and Time of Simulation 5 Years

cantly larger than Δx (see Figure 11). This indicates that the use of noncompatible Δx and Δt can cause the numerical solutions to oscillate around the exact solution violently. It may be explained as follows: The tridiagonal coefficient matrix is not diagonally dominant for the centered-in-space approximation and is not strongly diagonally dominant for the backward-in-space approximation. The grid spacing is too small to "absorb" numerically the injected mass of substance during the period between two time levels. Thus, the space and time increments should be chosen carefully and adapted to the problem.

A matrix is diagonally dominant if its main diagonal element of each row is at least as large as the sum of the absolute values of all the off-diagonal elements in that row, and it is strongly diagonally dominant if its main diagonal element of each row is significantly larger than the sum of the absolute values of all the off-diagonal elements in that row. A diagonally dominant matrix can avoid "ill-conditioning," yet a strongly diagonally dominant matrix can ensure that the solutions are convergent and the computations are stable with respect to the growth of rounding errors. The rate of convergence slows down as the Peclet number is increased.

Comparison of Three Approximations

In applying numerical methods, four general characteristics are usually of concern:

1. Accuracy, which deals with how well the discretized solution approximates the solution to the mathematical equation it represents.

2. Efficiency, which is a measure of how much computational work is required to obtain the solution.

3. Convergence, which concerns whether the solution is possible or not to approach the exact solution when Δx and Δt tend to zero, at a given distance and time.

4. Stability, which addresses the question of whether the differences between the numerical and exact solutions are bounded or not when time tends toward infinity, for a given Δx and Δt.

All three numerical approximations based on the Crank-Nicolson method meet the above criteria under most of the practical conditions. An accuracy of first-order in space is sufficient for most field problems. The Crank-Nicolson method is also efficient, since its time derivative analog is second-order correct and, thus, a larger time increment can be used. Overall, the centered-in-space approximation is more accurate but has a tendency to generate overshoot and oscillation. The backward-in-space approximation is more stable but tends to produce more numerical dispersion. The forward-in-space approximation is not suitable to use for solving the transport problems, since it tends to yield an "ill-conditioned" tridiagonal coefficient matrix. A matrix is said to be "ill-conditioned" if small changes in the elements of the matrix cause large changes in the solution of a system of equations. An "ill-conditioned" matrix can cause the solution to be numerically unstable and produce catastrophic rounding errors.

There is no practical reason for using the forward-in-space approximation to approximate the convection term in the transport equation except for comparison only. The centered-in-space approximation should only be used to discretize the convection term for small values of the Peclet number. When Peclet numbers are large, the noncentral difference approximation must be used. When the noncentral differences are used to approximate the first-space derivative in the convection term, the forward and backward differences are used according to whether the sign of the convection term is positive or negative. This method is called an "upstream" or "upwind" difference scheme. The forward difference is used when the convection term is positive; the backward difference is used when the convection term is negative. This ensures that the one-sided (noncentral) difference scheme is always on the "upstream" side of the point at which $\partial C/\partial t$ is evaluated. Since the convection term in the transport equation is always negative, the backward-in-space approximation should be used to discretize the first space derivative in order to obtain the "upstream" difference scheme and to ensure that the tridiagonal coefficient matrix is always diagonally dominant. The "upstream" difference scheme is stable for large Peclet numbers (see Figures 8 through 10). This stability is achieved because of the diagonal dominance of the coefficient matrix.

Computational difficulties were encountered for both forward- and centered-in-space approximations with large Peclet numbers. Numerical instability was experienced, as shown in Figures 8 through 12, when an infinite Peclet number was used, especially with small step sizes. The centered-in-space approximation exhibits a typically oscillatory nature for large Peclet numbers with a constant distance. When the Peclet number is increased or the step size is reduced, the central difference solutions deteriorate further, whereas the upstream difference solutions continue to be qualitatively correct.

It has been found that the solutions of the centered- and backward-in-space approximations converge to the exact solution as $\Delta x \to 0$ for a fixed Pe. However, when Pe is large, the centered-in-space solution oscillates around the exact solution, whereas the backward-in-space solution closely approximates the exact solution. The upstream difference solution approximated the exact solution very closely when $Pe \to \infty$ was discovered. However, the central difference scheme demonstrated an oscillatory behavior and diverged violently from the exact solution as $Pe \to \infty$ and $\Delta x \to 0$.

Even though the centered-in-space approximation is more accurate locally than the backward-in-space approximation, the overall effect is poor. The centered-in-space approximation turns out to be less accurate than the backward-in-space approximation, and it also requires a more strict stability condition for large values of the Peclet number. On the other hand, the backward-in-space approximation has been found to be fairly accurate and stable, especially for large values of the Peclet number. Nevertheless, since the backward-in-space approximation is only first-order accurate in space, the increase in the discretization (truncation) error from the backward-in-space approximation should be weighed against the better properties of its tridiagonal coefficient matrix.

In summary, the forward-in-space approximation is not appropriate for solving transport problems, and the use of the centered- or backward-in-space approximation may be dependent on transport parameters and coefficients, and the accuracy of the solution desired. However, the backward-in-space approximation is usually preferred.

CHAPTER V

MODEL APPLICATION

The previous chapter verified the numerical model that was developed. This chapter deals with the application of the model to predict the spatial and temporal concentration distributions of the leachate substances below a sanitary landfill under hypothetical conditions.

Evaluation of the Transport Properties

The soil is a complex geological formation. Soil profiles often show differences in texture and structure with depth. The differences in texture, structure, and hydrogeologic parameters--such as porosity, moisture content, soil density, and fluid viscosity of a soil-water system--can be interrelated in affecting the movement of the leachate. The character of the soils, especially the surface character of the minerals, affects the distribution coefficient and the adsorption. The phenomena of the saturated and unsaturated flow also affect the leachate movement.

The dispersion-diffusion phenomena in porous media are often extremely complicated due to the infinite number of possible pore geometries. These pore geometries are not included in the mathematical analysis because of the complexity and they can only be described by characteristic properties of the medium itself.

The convection is dependent on the average pore-water or seepage velocity. The seepage velocity (V), or the permeability, may vary with time and effects of chemical wastes. The clay liner can be affected by chemical wastes--especially strong acids, bases, and organic chemicals--and increases in the permeability. Even if the liner has a permeability approaching zero at the beginning, diffusion of the leachate will eventually cause the liner to leak. A few large pores or cracks in the liner will result in a higher permeability.

The transformation process represents changes of the substance concentration due to physical, chemical, and biological reactions during the transport of the leachate in the soil-water system. This may include irreversible adsorption, precipitation, and ion exchange. The transformation coefficient (P), or the rate of the substance removed from the solution into the soil matrix, is dependent on the geophysical and geochemical properties of the soil and on the interaction between the soil and the leachate. The value of P is usually small and increases with increases in reactive materials content in the soil-water system.

The distribution coefficient (K) is a measure of the ratio of the adsorbed solute concentration to the solute concentration in the solution. The value of K is determined in the laboratory by shaking leachate containing a known concentration of substance with soil until equilibrium is reached and then measuring the substance concentration in the supernatant. The ratio of the adsorbed concentration (S) to the concentration in the supernatant (C) is equal to K for a linear equilibrium adsorption isotherm. The distribution coefficient is also dependent on properties of the soil-water system.

When media properties alternate randomly in space, mean values of the transport parameters and coefficients can be used. However, when distinct regions of different media properties can be distinguished in the media, the media should be separated into different regions or layers. Each layer is considered to be homogeneous and isotropic, and has constant transport properties. If the layers are thin, the geological formation approximates a nonhomogeneous and anisotropic medium. The clay liner has a distinct property from the soil media and, thus, should be considered as a separate layer.

Because of the slow movement of the leachate, an artificial lower boundary is selected to study the spatial and temporal distributions of the leachate substances. This artificial boundary is located a sufficient distance from the project area as to have a negligible effect on the area of interest during the simulation period. Although the lower-boundary condition is arbitrary, the influence of the artificial boundary (or the simulation distance) is checked by comparing the results of several different locations of the artificial boundary. The best results are locations where the solute concentration front cannot reach and the solute concentration is zero. However, for small concentration gradients, the effect of the artificial boundary on the concentration distribution is negligible.

Hypothetical Conditions of the Site

Some hypothetical conditions are made in order to clarify the application of the model. There are no floodplains located on the landfill site. The only water that comes in contact with the landfills are precipitation and surface runoff on the cells. There are no water tables above the bottom of the landfill cells. The leachate flow is in a one-dimensional downward direction and the amount of leachate passing through the sidewalls is negligible. The source of the leachate substances is considered as a point source and is located in the bottom of the cells.

A diagrammatic sketch of a landfill cell is shown in Figure 13. The soils below the landfill cells have transport parameters of $D' = 1.0$ m^2/year, $V' = 1.0$ m/year, and $P' = 0.1$ $year^{-1}$. A preventive measure against any potential health hazards is to install a clay-type material liner in the bottom and on the sidewalls of the cells during construction of the cells. The liner is considered as a separate soil layer and has transport parameters of $D' = 0.1$ m^2/year, $V' = 0.1$ m/year, and $P' = 0.1$ $year^{-1}$.

Application of the Model

Two different types of landfills, one without a liner and one with a liner, are adopted to study the effects of a liner on the leachate concentration distribution. The design thickness of the liner is 1.0 meter and the decay coefficient of the leachate source concentration is 0.05 $year^{-1}$. A space increment of 0.1 meter and a time increment of 1.0 year are used. The distance of study (or the combined thickness of layers) is 20 meters and the simulation period is 100 years. The location of the artificial lower boundary (or the simulation distance) is 30 meters away from the source. However, the artificial lower boundary for the liner is only 3 meters from the source. The output data are listed in Appendix D. Note that all three numerical approximations give about the same results. The results are presented in Figures 14 and 16 for media without a liner (refer to Test Data No. 1) and in Figures 15 and 17 for media with a liner (refer to Test Data No. 2).

Figures 14 and 15 show the spatial concentration distributions at time periods of 10, 20, and 50 years for media without a liner and media

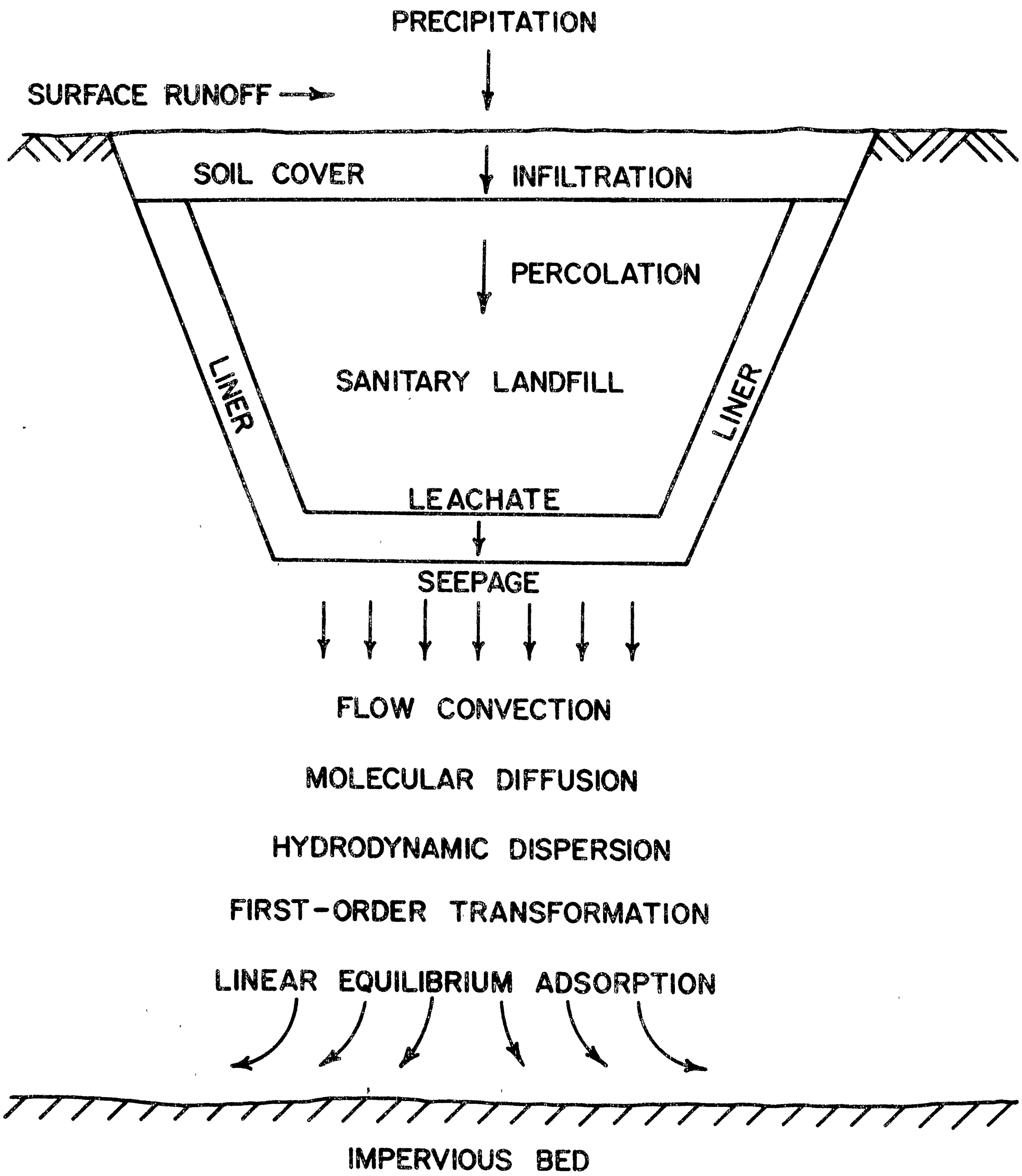

Figure 13. Diagrammatic Sketch of a Landfill Cell

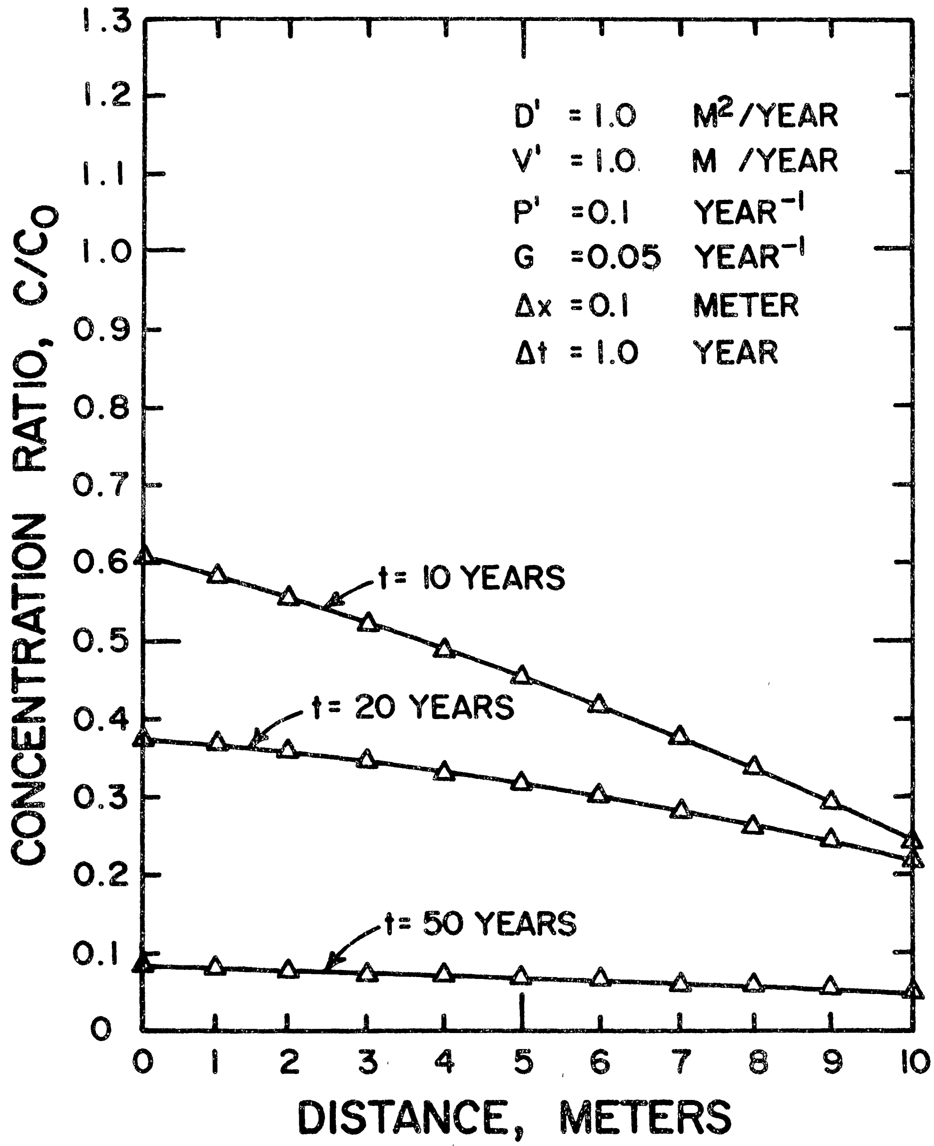

Figure 14. Spatial Concentration Distributions in Single-Layered Soil Media (Without Liner) After 10, 20, and 50 Years

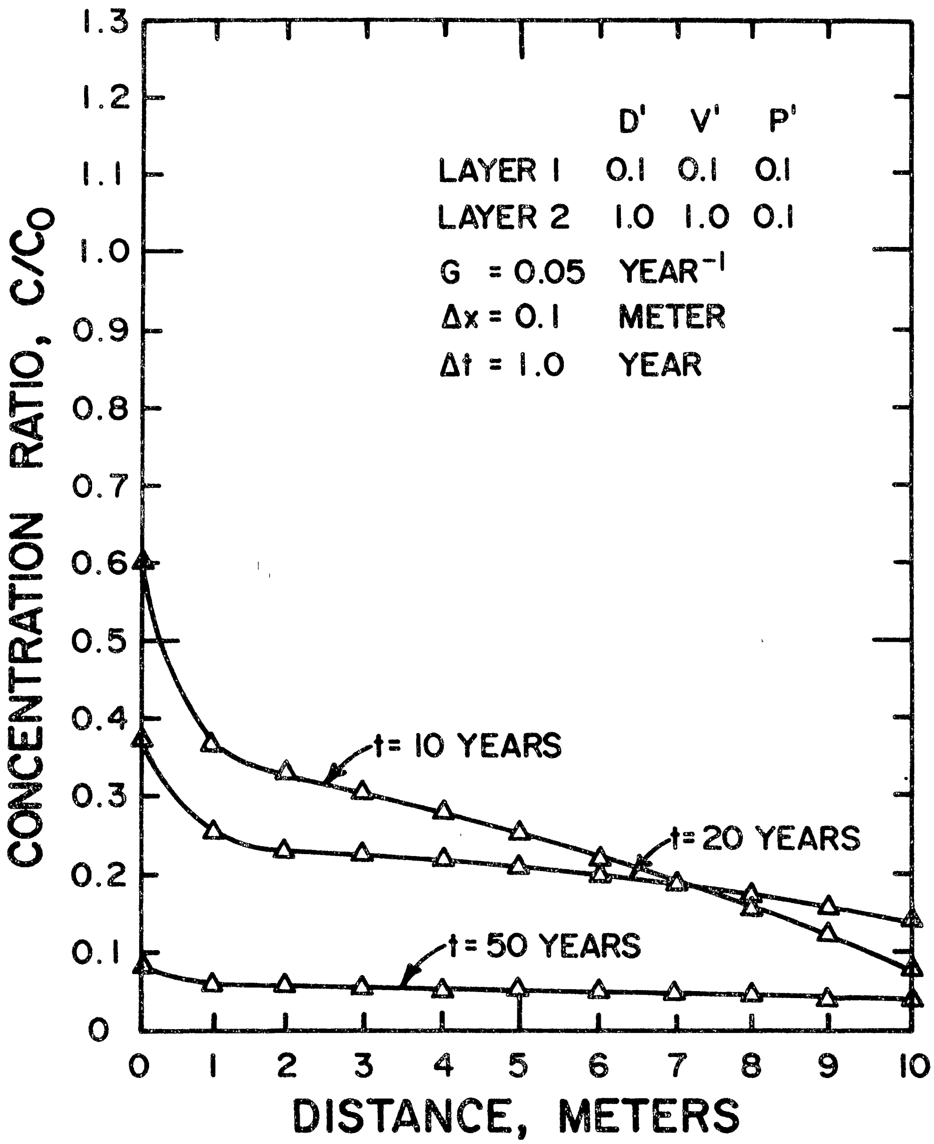

Figure 15. Spatial Concentration Distributions in Two-Layered Soil Media (With Liner) After 10, 20, and 50 Years

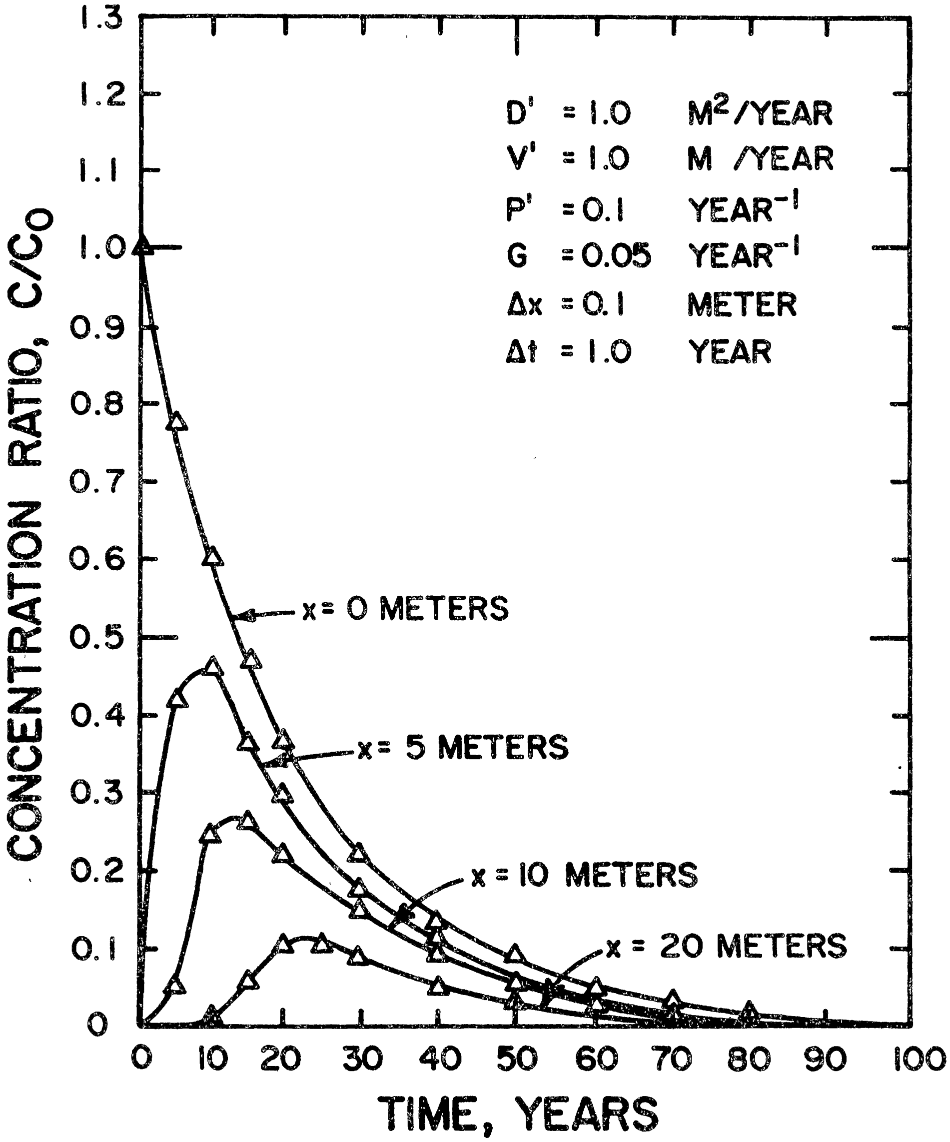

Figure 16. Temporal Concentration Distributions in Single-Layered Soil Media (Without Liner) at Distances 0, 5, 10, and 20 Meters From Source

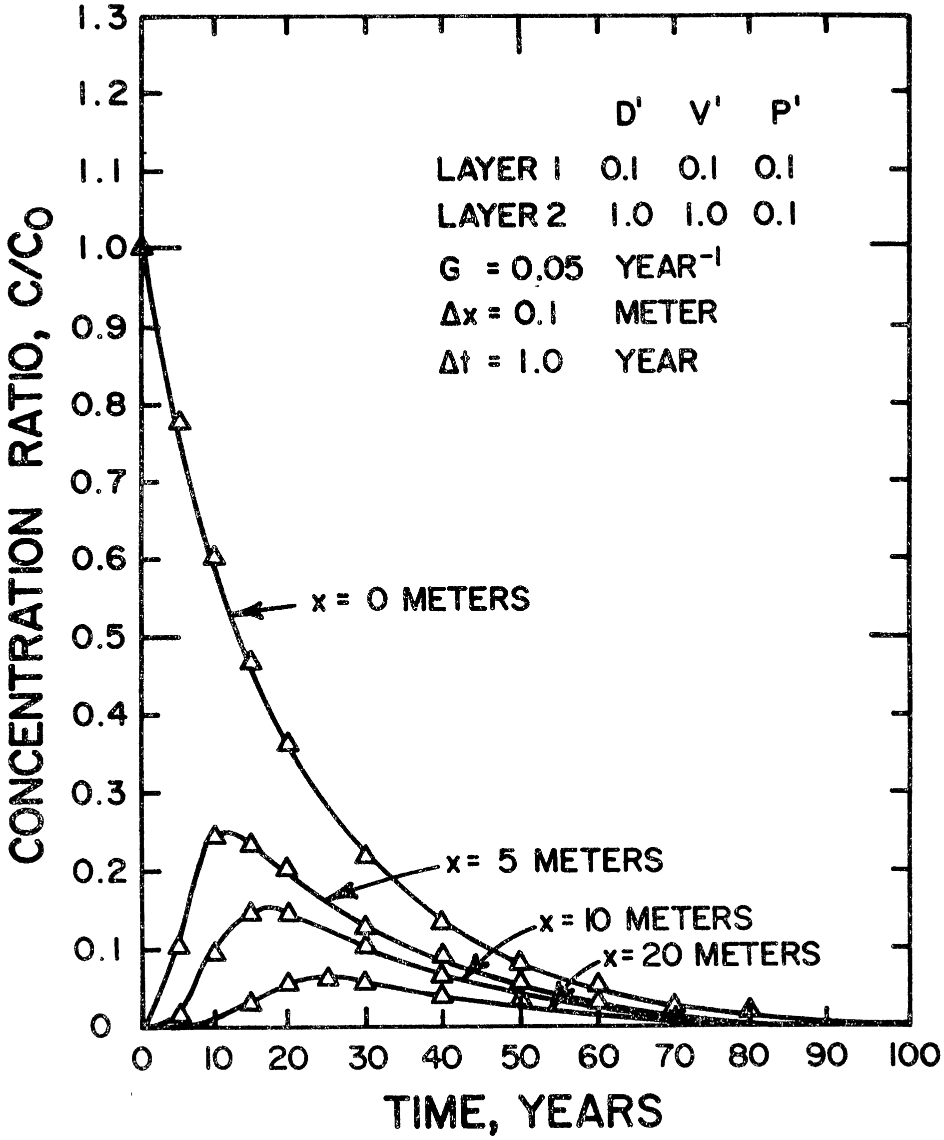

Figure 17. Temporal Concentration Distributions in Two-Layered Soil Media (With Liner) at Distances 0, 5, 10, and 20 Meters From Source

with a liner, respectively. Note that the decay of the source causes the concentration profiles to approach zero with increase in time. If the source is constant, the concentration profiles will approach one, which is equal to the source concentration, with increase in time. Figure 15 indicates the liner can dramatically reduce the leachate substances that pass through.

Figures 16 and 17 present the temporal concentration distributions at distances of 0, 5, 10, and 20 meters for media without a liner and media with a liner, respectively. The peaks of the curves represent the maximum concentrations at different locations. The peaks occurred at different times for different locations. It is interesting to note that, in this case, a liner one meter thick can reduce the concentration to about one-half of media not containing a liner.

The movement of the leachate substances in soils depends upon the combined effects of dispersion-diffusion, convection, adsorption, and transformation: larger dispersion-diffusion increases substance spread; larger seepage velocity increases substance movement; and larger transformation decreases substance movement. With consideration of the effect of adsorption, dispersion-diffusion, seepage velocity, and transformation can be reduced.

The accuracy of a predicted concentration distribution depends primarily on the accuracy of the transport parameters and coefficients used in the simulation process. Thus, more effort may be needed to quantify various parameters and coefficients rather than to define and construct the mathematical equations. The results will not be useful if measures of the transport parameters and coefficients are not adequate, even though the model is appropriate for describing the leachate migration process. The major obstacle in applying the model in a useful manner for the solutions of field problems is often not the computational difficulty but, rather, deficiencies in measuring the appropriate transport parameters and coefficients for the model input.

CHAPTER VI

SUMMARY AND CONCLUSIONS

Modeling of leachate migration is an essential step in determining the impact of waste-disposal facilities on surface and subsurface water quality. A computer model written in WATFIV has been developed for simulating the leachate transport below a sanitary landfill or waste-disposal site. The model is capable of simulating the movement of leachate substances through single-layered/multilayered soil media. Four major mechanisms--dispersion-diffusion, convection, linear equilibrium adsorption, and first-order transformation--are considered in the transport process.

A finite difference technique based on the Crank-Nicolson method is used to approximate the governing transport Equation (3.11) and to generate a system of algebraic equations. The Gauss elimination algorithm is used as a direct method to solve the resulting tridiagonal system of linear algebraic equations. Three finite difference approximations for the transport convection term are applied to compare the outcomes.

The model developed in this study is flexible and practical, and accounts for major transport processes. The model can be adapted to other field problems to estimate spatial and temporal distributions of substance concentrations and time of travel between a substance source and a groundwater sink (a discharge point such as a well, river, stream, creek, or spring). The model can also be used as a tool for water-quality studies, waste-disposal site selection, evaluation of environmental impact, monitoring of pollutant movement underground, and design of projects to minimize groundwater contamination.

The performance of the numerical model is evaluated by comparing its results with one-dimensional analytical solutions in a single-layered soil medium. The model is applied to a sanitary landfill, based on hypothetical conditions, to predict the spatial and temporal concentration distributions of the leachate substances.

Based on the results obtained from the verification and application of the model, the following conclusions are presented:

1. The Crank-Nicolson method with the centered-in-space approximation is more accurate but tends to generate overshoot and oscillation.

2. The Crank-Nicolson method with the backward-in-space approximation is more stable but tends to produce more numerical dispersion.

3. The Crank-Nicolson method with the forward-in-space approximation is the reverse condition of the Crank-Nicolson method with the backward-in-space approximation. It is less stable and reduces the dispersion effect. The forward-in-space approximation should only be used when the convection term is positive, which will ensure that the tridiagonal coefficient matrix is not "ill-conditioned."

4. For multilayered soil media, the order of the layers does not affect the effluent concentration. If several layers have the same transport properties, the lengths or thicknesses of these individual layers can be added and used as a single equivalent layer.

5. The accuracy of the model is dependent on space and time increments and on the distance of the artificial lower boundary that was

chosen. With small space and time increments and adequate simulation distance, the results of the numerical model are in close agreement with the analytical model.

6. The average CPU time of solving a 100 space steps by 100 time steps is about 2.1 seconds for three numerical solutions and costs about one dollar. The cost can be significantly reduced for running a single numerical solution. Thus, the model is very efficient and economical.

Although the model developed in this study is efficient and easy to use for solving transport problems, the following recommendations for further study are made:

1. Test the model with available field data.

2. Conduct research to quantify various transport parameters and coefficients.

3. Upgrade the model to account for line and area sources.

4. Extend the present one-dimensional model to a higher dimensional model.

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APPENDIX A

DERIVATION OF FINITE DIFFERENCE EQUATIONS

The finite difference formulation is based on the truncated Taylor series, which are similar to the definition of the derivatives they approximate. The Taylor series expansion for C_i^{n+1} and C_i^n at time levels n+1 and n, respectively, about the point $C_i^{n+\frac{1}{2}}$ at time level $n+\frac{1}{2}$ are expressed as

$$C_i^{n+1} = C_i^{n+\frac{1}{2}} + \frac{\Delta t}{2}\frac{\partial C_i^{n+\frac{1}{2}}}{\partial t} + \frac{\Delta t^2}{8}\frac{\partial^2 C_i^{n+\frac{1}{2}}}{\partial t^2} + \frac{\Delta t^3}{48}\frac{\partial^3 C_i^{n+\frac{1}{2}}}{\partial t^3} + \ldots \qquad (A.1)$$

and

$$C_i^{n} = C_i^{n+\frac{1}{2}} - \frac{\Delta t}{2}\frac{\partial C_i^{n+\frac{1}{2}}}{\partial t} + \frac{\Delta t^2}{8}\frac{\partial^2 C_i^{n+\frac{1}{2}}}{\partial t^2} - \frac{\Delta t^3}{48}\frac{\partial^3 C_i^{n+\frac{1}{2}}}{\partial t^3} \pm \ldots \qquad (A.2)$$

Equations (A.1) and (A.2) can be solved together to give

$$\frac{C_i^{n+1} + C_i^n}{2} = C_i^{n+\frac{1}{2}} + \frac{\Delta t^2}{8}\frac{\partial^2 C_i^{n+\frac{1}{2}}}{\partial t^2} + \ldots \qquad (A.3)$$

and

$$\frac{C_i^{n+1} - C_i^n}{\Delta t} = \frac{\partial C_i^{n+\frac{1}{2}}}{\partial t} + \frac{\Delta t^2}{24}\frac{\partial^3 C_i^{n+\frac{1}{2}}}{\partial t^3} + \ldots \qquad (A.4)$$

where Δt represents the time increment; and i, n denote the space position and time level, respectively.

Similarly, the Taylor series expansion for C_{i+1}^{n+1} and C_{i-1}^{n+1} at space positions i+1 and i-1, respectively, about the point C_i^{n+1} at space position i, and for C_{i+1}^n and C_{i-1}^n about the point C_i^n are

$$C_{i+1}^{n+1} = C_i^{n+1} + \Delta x\,\frac{\partial C_i^{n+1}}{\partial x} + \frac{\Delta x^2}{2}\,\frac{\partial^2 C_i^{n+1}}{\partial x^2} + \frac{\Delta x^3}{6}\frac{\partial^3 C_i^{n+1}}{\partial x^3} + \ldots \qquad (A.5)$$

$$C_{i-1}^{n+1} = C_i^{n+1} - \Delta x \frac{\partial C_i^{n+1}}{\partial x} + \frac{\Delta x^2}{2} \frac{\partial^2 C_i^{n+1}}{\partial x^2} - \frac{\Delta x^3}{6} \frac{\partial^3 C_i^{n+1}}{\partial x^3} \pm \ldots \quad (A.6)$$

$$C_{i+1}^{n} = C_i^{n} + \Delta x \frac{\partial C_i^{n}}{\partial x} + \frac{\Delta x^2}{2} \frac{\partial^2 C_i^{n}}{\partial x^2} + \frac{\Delta x^3}{6} \frac{\partial^3 C_i^{n}}{\partial x^3} + \ldots \quad (A.7)$$

and

$$C_{i-1}^{n} = C_i^{n} - \Delta x \frac{\partial C_i^{n}}{\partial x} + \frac{\Delta x^2}{2} \frac{\partial^2 C_i^{n}}{\partial x^2} - \frac{\Delta x^3}{6} \frac{\partial^3 C_i^{n}}{\partial x^3} \pm \ldots \quad (A.8)$$

Equations (A.5), (A.6), (A.7), and (A.8) can be rearranged as

$$\frac{C_{i+1}^{n+1} - C_i^{n+1}}{\Delta x} = \frac{\partial C_i^{n+1}}{\partial x} + \frac{\Delta x}{2} \frac{\partial^2 C_i^{n+1}}{\partial x^2} + \frac{\Delta x^2}{6} \frac{\partial^3 C_i^{n+1}}{\partial x^3} + \ldots \quad (A.9)$$

$$\frac{C_{i}^{n+1} - C_{i-1}^{n+1}}{\Delta x} = \frac{\partial C_i^{n+1}}{\partial x} - \frac{\Delta x}{2} \frac{\partial^2 C_i^{n+1}}{\partial x^2} + \frac{\Delta x^2}{6} \frac{\partial^3 C_i^{n+1}}{\partial x^3} \mp \ldots \quad (A.10)$$

$$\frac{C_{i+1}^{n} - C_i^{n}}{\Delta x} = \frac{\partial C_i^{n}}{\partial x} + \frac{\Delta x}{2} \frac{\partial^2 C_i^{n}}{\partial x^2} + \frac{\Delta x^2}{6} \frac{\partial^3 C_i^{n}}{\partial x^3} + \ldots \quad (A.11)$$

and

$$\frac{C_{i}^{n} - C_{i-1}^{n}}{\Delta x} = \frac{\partial C_i^{n}}{\partial x} - \frac{\Delta x}{2} \frac{\partial^2 C_i^{n}}{\partial x^2} + \frac{\Delta x^2}{6} \frac{\partial^3 C_i^{n}}{\partial x^3} \mp \ldots \quad (A.12)$$

Combining Equations (A.9) and (A.10), and Equations (A.11) and (A.12) to obtain

$$\frac{C_{i+1}^{n+1} - C_{i-1}^{n+1}}{2\Delta x} = \frac{\partial C_i^{n+1}}{\partial x} + \frac{\Delta x^2}{6} \frac{\partial^3 C_i^{n+1}}{\partial x^3} + \ldots \quad (A.13)$$

and

$$\frac{C_{i+1}^{n} - C_{i-1}^{n}}{2\Delta x} = \frac{\partial C_i^{n}}{\partial x} + \frac{\Delta x^2}{6} \frac{\partial^3 C_i^{n}}{\partial x^3} + \ldots \quad (A.14)$$

If Equation (A.10) is subtracted from Equation (A.9) and then divided by Δx, the result is

$$\frac{C_{i+1}^{n+1} - 2C_i^{n+1} + C_{i-1}^{n+1}}{\Delta x^2} = \frac{\partial^2 C_i^{n+1}}{\partial x^2} + \frac{\Delta x^2}{12}\frac{\partial^4 C_i^{n+1}}{\partial x^4} + \ldots \tag{A.15}$$

Similarly, if Equation (A.12) is subtracted from Equation (A.11) and then divided by Δx, the result is

$$\frac{C_{i+1}^{n} - 2C_i^{n} + C_{i-1}^{n}}{\Delta x^2} = \frac{\partial^2 C_i^{n}}{\partial x^2} + \frac{\Delta x^2}{12}\frac{\partial^4 C_i^{n}}{\partial x^4} + \ldots \tag{A.16}$$

Using the principle of Equation (A.3), the following equations can be derived from Equations (A.9) through (A.16):

$$\begin{aligned}
&\frac{1}{2}\left[\frac{C_{i+1}^{n+1} - 2C_i^{n+1} + C_{i-1}^{n+1}}{\Delta x^2} + \frac{C_{i+1}^{n} - 2C_i^{n} + C_{i-1}^{n}}{\Delta x^2}\right] \\
&\quad = \left\{\frac{\partial^2 C_i^{n+\frac{1}{2}}}{\partial x^2} + \frac{\Delta x^2}{12}\frac{\partial^4 C_i^{n+\frac{1}{2}}}{\partial x^4} + \ldots\right\} \\
&\quad + \frac{\Delta t^2}{8}\frac{\partial^2}{\partial t^2}\left\{\frac{\partial^2 C_i^{n+\frac{1}{2}}}{\partial x^2}\right. \\
&\quad \left. + \frac{\Delta x^2}{12}\frac{\partial^4 C_i^{n+\frac{1}{2}}}{\partial x^4} + \ldots\right\} + \ldots
\end{aligned} \tag{A.17}$$

$$\begin{aligned}
&\frac{1}{2}\left[\frac{C_{i+1}^{n+1} - C_{i-1}^{n+1}}{2\Delta x} + \frac{C_{i+1}^{n} - C_{i-1}^{n}}{2\Delta x}\right] \\
&\quad = \left\{\frac{\partial C_i^{n+\frac{1}{2}}}{\partial x} + \frac{\Delta x^2}{6}\frac{\partial^3 C_i^{n+\frac{1}{2}}}{\partial x^3} + \ldots\right\}
\end{aligned}$$

$$+ \frac{\Delta t^2}{8} \frac{\partial^2}{\partial t^2} \left\{ \frac{\partial C_i^{n+\frac{1}{2}}}{\partial x} + \frac{\Delta x^2}{6} \frac{\partial^3 C_i^{n+\frac{1}{2}}}{\partial x^3} + \ldots \right\} + \ldots \tag{A.18}$$

$$\frac{1}{2}\left[\frac{C_i^{n+1} - C_{i-1}^{n+1}}{\Delta x} + \frac{C_i^n - C_{i-1}^n}{\Delta x}\right]$$

$$= \left\{ \frac{\partial C_i^{n+\frac{1}{2}}}{\partial x} - \frac{\Delta x}{2} \frac{\partial^2 C_i^{n+\frac{1}{2}}}{\partial x^2} + \frac{\Delta x^2}{6} \frac{\partial^3 C_i^{n+\frac{1}{2}}}{\partial x^3} \mp \ldots \right\}$$

$$+ \frac{\Delta t^2}{8} \frac{\partial^2}{\partial t^2} \left\{ \frac{\partial C_i^{n+\frac{1}{2}}}{\partial x} - \frac{\Delta x}{2} \frac{\partial^2 C_i^{n+\frac{1}{2}}}{\partial x^2} + \frac{\Delta x^2}{6} \frac{\partial^3 C_i^{n+\frac{1}{2}}}{\partial x^3} \mp \ldots \right\} + \ldots \tag{A.19}$$

and

$$\frac{1}{2}\left[\frac{C_{i+1}^{n+1} - C_i^{n+1}}{\Delta x} + \frac{C_{i+1}^n - C_i^n}{\Delta x}\right]$$

$$= \left\{ \frac{\partial C_i^{n+\frac{1}{2}}}{\partial x} + \frac{\Delta x}{2} \frac{\partial^2 C_i^{n+\frac{1}{2}}}{\partial x^2} + \frac{\Delta x^2}{6} \frac{\partial^3 C_i^{n+\frac{1}{2}}}{\partial x^3} + \ldots \right\}$$

$$+ \frac{\Delta t^2}{8} \frac{\partial^2}{\partial t^2} \left\{ \frac{\partial C_i^{n+\frac{1}{2}}}{\partial x} + \frac{\Delta x}{2} \frac{\partial^2 C_i^{n+\frac{1}{2}}}{\partial x^2} + \frac{\Delta x^2}{6} \frac{\partial^3 C_i^{n+\frac{1}{2}}}{\partial x^3} + \ldots \right\} + \ldots \tag{A.20}$$

The governing transport Equation (3.11) in terms of $C_i^{n+\frac{1}{2}}$ is written as

$$\frac{\partial C_i^{n+\frac{1}{2}}}{\partial t} = D' \frac{\partial^2 C_i^{n+\frac{1}{2}}}{\partial x^2} - V' \frac{\partial C_i^{n+\frac{1}{2}}}{\partial x} - P' C_i^{n+\frac{1}{2}} \qquad (A.21)$$

Rearranging Equations (A.3), (A.4), (A.17), and (A.18) and then substituting in Equation (A.21)

$$\left\{ \frac{C_i^{n+1} - C_i^n}{\Delta t} - \frac{\Delta t^2}{24} \frac{\partial^3 C_i^{n+\frac{1}{2}}}{\partial t^3} - \ldots \right\}$$

$$= D' \left\{ \frac{1}{2} \left[\frac{C_{i+1}^{n+1} - 2C_i^{n+1} + C_{i-1}^{n+1}}{\Delta x^2} + \frac{C_{i+1}^n - 2C_i^n + C_{i-1}^n}{\Delta x^2} \right] \right.$$

$$- \left[\frac{\Delta x^2}{12} \frac{\partial^4 C_i^{n+\frac{1}{2}}}{\partial x^4} + \ldots \right] - \frac{\Delta t^2}{8} \frac{\partial^2}{\partial t^2} \left[\frac{\partial^2 C_i^{n+\frac{1}{2}}}{\partial x^2} \right.$$

$$\left. \left. + \frac{\Delta x^2}{12} \frac{\partial^4 C_i^{n+\frac{1}{2}}}{\partial x^4} + \ldots \right] - \ldots \right\}$$

$$- V' \left\{ \frac{1}{2} \left[\frac{C_{i+1}^{n+1} - C_{i-1}^{n+1}}{2\Delta x} + \frac{C_{i+1}^n - C_{i-1}^n}{2\Delta x} \right] - \left[\frac{\Delta x^2}{6} \frac{\partial^3 C_i^{n+\frac{1}{2}}}{\partial x^3} + \ldots \right] \right.$$

$$\left. - \frac{\Delta t^2}{8} \frac{\partial^2}{\partial t^2} \left[\frac{\partial C_i^{n+\frac{1}{2}}}{\partial x} + \frac{\Delta x^2}{6} \frac{\partial^3 C_i^{n+\frac{1}{2}}}{\partial x^3} + \ldots \right] - \ldots \right\}$$

$$- P' \left\{ \frac{C_i^{n+1} + C_i^n}{2} - \frac{\Delta t^2}{8} \frac{\partial^2 C_i^{n+\frac{1}{2}}}{\partial t^2} - \ldots \right\} \qquad (A.22)$$

If all the derivative terms in Equation (A.22) are omitted, the equation becomes

$$\frac{C_i^{n+1} - C_i^n}{\Delta t} = D' \left(\frac{1}{2}\right) \left[\frac{C_{i+1}^{n+1} - 2C_i^{n+1} + C_{i-1}^{n+1}}{\Delta x^2} + \frac{C_{i+1}^n - 2C_i^n + C_{i-1}^n}{\Delta x^2} \right]$$

$$- V' \left(\frac{1}{2}\right) \left[\frac{C_{i+1}^{n+1} - C_{i-1}^{n+1}}{2\Delta x} + \frac{C_{i+1}^{n} - C_{i-1}^{n}}{2\Delta x}\right]$$

$$- P' \left(\frac{1}{2}\right) [C_i^{n+1} + C_i^n] \tag{A.23}$$

Equation (A.23) represents the finite difference equation for the Crank-Nicolson method with the centered-in-space approximation to the convection term.

In a similar manner, the finite difference equation for the Crank-Nicolson method with the backward-in-space approximation to the convection term can be obtained from Equations (A.3), (A.4), (A.17), and (A.19):

$$\frac{C_i^{n+1} - C_i^n}{\Delta t} = D' \left(\frac{1}{2}\right) \left[\frac{C_{i+1}^{n+1} - 2C_i^{n+1} + C_{i-1}^{n+1}}{\Delta x^2} + \frac{C_{i+1}^{n} - 2C_i^{n} + C_{i-1}^{n}}{\Delta x^2}\right]$$

$$- V' \left(\frac{1}{2}\right) \left[\frac{C_{i}^{n+1} - C_{i-1}^{n+1}}{\Delta x} + \frac{C_{i}^{n} - C_{i-1}^{n}}{\Delta x}\right]$$

$$- P' \left(\frac{1}{2}\right) [C_i^{n+1} + C_i^n] \tag{A.24}$$

and the finite difference equation for the Crank-Nicolson method with the forward-in-space approximation to the convection term can be obtained from Equations (A.3), (A.4), (A.17), and (A.20):

$$\frac{C_i^{n+1} - C_i^n}{\Delta t} = D' \left(\frac{1}{2}\right) \left[\frac{C_{i+1}^{n+1} - 2C_i^{n+1} + C_{i-1}^{n+1}}{\Delta x^2} + \frac{C_{i+1}^{n} - 2C_i^{n} + C_{i-1}^{n}}{\Delta x^2}\right]$$

$$- V' \left(\frac{1}{2}\right) \left[\frac{C_{i+1}^{n+1} - C_{i}^{n+1}}{\Delta x} + \frac{C_{i+1}^{n} - C_{i}^{n}}{\Delta x}\right]$$

$$- P' \left(\frac{1}{2}\right) [C_i^{n+1} + C_i^n] \tag{A.25}$$

Note that all three numerical schemes have the same formulation for both dispersion and transformation terms.

APPENDIX B

COMPUTER PROGRAM LISTING

```
C
C
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C
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C                      ------------------------------
C                     | LEACHATE TRANSPORT MODELING  |
C                     | PROGRAMMER:    WEI-MING WANG |
C                     | STUDENT ID:        294889    |
C                     | SCHOOL OF CIVIL ENGINEERING  |
C                     | OKLAHOMA  STATE  UNIVERSITY  |
C                     |         JULY,  1985          |
C                     |                              |
C                      ------------------------------
C
C
C     STATEMENT
C     ANALYTICAL AND FINITE DIFFERENCE NUMERICAL SOLUTIONS FOR LEACHATE
C     TRANSPORT EQUATION WITH DISPERSION-DIFFUSION, CONVECTION, LINEAR
C     EQUILIBRIUM ADSORPTION, AND FIRST-ORDER TRANSFORMATION CONSIDERED
C     THE NUMERICAL SOLUTIONS ARE BASED ON CRANK-NICOLSON METHOD WITH
C     CENTERED-, BACKWARD-, AND FORWARD-IN-SPACE APPROXIMATIONS
C     INITIAL CONDITION: C(X,O)=O, FOR X > O
C     UPPER BOUNDARY CONDITION: C(O,T)=CO*EXP(-G*T)
C     LOWER BOUNDARY CONDITION: DC(XL,T)/DX=O
C     NOTATIONS:
C      T = TIME (YEAR)
C      X = DISTANCE OR SPACE (METER)
C      C = SOLUTE CONCENTRATION (MASS/VOLUME)
C      D = EFFECTIVE DISPERSION COEFFICIENT (SQUARE METER/YEAR)
C      V = EFFECTIVE AVERAGE PORE-WATER OR SEEPAGE VELOCITY (METER/YEAR)
C      P = EFFECTIVE TRANSFORMATION COEFFICIENT (1/YEAR)
C      G = DECAY COEFFICIENT FOR THE SOURCE OF SUBSTANCE (1/YEAR)
C      K = INDEX OF SOLUTION METHODS
C      B = RIGHT-HAND-SIDE COLUMN MATRIX ELEMENTS
C     CO = INITIAL CONCENTRATION AT X=O, T=O (MASS/VOLUME)
C     DX = SPACE INCREMENT OR STEP-SIZE
C     DT = TIME INCREMENT OR STEP-SIZE
C     ID = INPUT DATA SETS IDENTIFICATION NUMBER
C     IX = INDEX OF SPACE STEPS
C     IT = INDEX OF TIME STEPS
C     NA = LAST STEP OF A LAYER AT INTERBOUNDARY OR LOWER BOUNDARY
C     NB = FIRST STEP OF A LAYER AT UPPER BOUNDARY OR INTERBOUNDARY
C     NL = NUMBER OF SOIL LAYERS
C     NX = TOTAL NUMBER OF SPACE STEPS OCCUPIED BY ALL LAYERS
C          OR NUMBER OF SPACE STEPS USED FOR SIMULATION TO A LAYER
C     NT = NUMBER OF TIME STEPS USED FOR SIMULATION
C     XL = DISTANCE OF SIMULATION TO A LAYER
C     XX = COMBINED THICKNESS OF ALL LAYERS
C     TT = TIME PERIOD OF SIMULATION
C     NBP1 = NB+1 = SECOND SPACE STEP OF A LAYER
C     NBP2 = NB+2 = THIRD SPACE STEP OF A LAYER
C     NXP1 = NX+1 = LAST SPACE STEP OF A SIMULATION DISTANCE
C     NTP1 = NT+1 = LAST TIME STEP OF A SIMULATION TIME
C     NSSL = NUMBER OF SPACE STEPS OCCUPIED BY A LAYER
C     NXOUT = SPACE STEPS FOR EACH PRINTOUT
C     NTOUT = TIME STEPS FOR EACH PRINTOUT
C     THICK = THICKNESS OF A LAYER
C     A1, A2, A3 = TRIDIAGONAL ELEMENTS ON LEFT-HAND-SIDE FOR NEXT
C                  TIME STEP
C     B1, B2, B3 = COEFFICIENTS ON RIGHT-HAND-SIDE FOR CURRENT TIME
C                  STEP
```

```
C      NOTES:
C        1. ANALYTICAL SOLUTION ONLY AVAILABLE FOR A SINGLE LAYER; BUT
C           NOT AVAILABLE FOR A SINGLE LAYER WITH EFFECTIVE DISPERSION
C           COEFFICIENT D < OR = 0, OR H1=V*V+4.*D*(P-G) < 0.
C        2. THE MAXIMUM NUMBER OF SPACE STEPS (NX) AND TIME STEPS (NT)
C           MUST BE ONE LESS THAN THE DIMENSIONS ASSIGNED.
C        3. INPUT DATA SETS CAN BE CONTINUOUSLY READ IN. READ IN GENERAL
C           INFORMATION FIRST AND THEN FOLLOWED BY EACH SOIL LAYER'S
C           CHARACTERISTICS.
C        4. IF CO=1 IS USED THE SOLUTIONS WOULD BE THE SAME AS THE RATIO
C           OF CURRENT CONCENTRATION AT DISTANCE X, TIME T, TO INITIAL
C           CONCENTRATION (CO) AT X=0, T=0.
C
C_____________________________________________________________________
C
C
C      MAIN PROGRAM:
       IMPLICIT REAL*8 (A-H,O-Z)
       COMMON D,V,P,G,DX,DT,NX,NT,NXP1,NTP1,NA,NB,NBP1,NBP2
       DIMENSION  T(201), X(501), C(4,201,501)
       ID=0
C      READ IN NUMBER OF SOIL LAYERS (NL), TOTAL NUMBER OF SPACE STEPS
C      OCCUPIED BY ALL LAYERS (NX), NUMBER OF TIME STEPS FOR SIMULATION
C      (NT), NUMBER OF SPACE AND TIME STEPS FOR EACH PRINTOUT (NXOUT),
C      (NTOUT), COMBINED THICKNESS OF ALL LAYERS (XX), TIME PERIOD OF
C      SIMULATION (TT), INITIAL CONCENTRATION (CO) AT X=0, T=0, AND THE
C      DECAY COEFFICIENT FOR THE SOURCE OF SUBSTANCE (G).
 11111 READ(5,*,END=99999)  NL,NX,NT,NXOUT,NTOUT,XX,TT,CO,G
       NN=NL
       ID=ID+1
       DX=XX/NX
       DT=TT/NT
       NXP1=NX+1
       NTP1=NT+1
       DO 10 IX=1, NXP1
 10    X(IX)=DX*(IX-1)
       DO 20 IT=1, NTP1
 20    T(IT)=DT*(IT-1)
       PRINT 100,  ID,NL,NX,NT,NXOUT,NTOUT,DX,DT,XX,TT,CO,G
       NA=1
       DO 90 IL=1, NL
C      READ IN NUMBER OF SPACE STEPS OCCUPIED BY A SOIL LAYER (NSSL).
C      NUMBER OF SPACE STEPS FOR SIMULATION TO A SOIL LAYER (NX),
C      EFFECTIVE DISPERSION COEFFICIENT (D), EFFECTIVE AVERAGE PORE-WATER
C      VELOCITY (V), AND EFFECTIVE TRANSFORMATION COEFFICIENT (P).
       READ,  NSSL,NX,D,V,P
       XL=DX*NX
       NXP1=NX+1
       THICK=DX*NSSL
       PRINT 200,  IL,D,V,P,THICK,XL
       NA=NA+NSSL
       NB=NA-NSSL
       NBP1=NB+1
       NBP2=NB+2
       DO 90 K=1, 4
C      K=1, FOR ANALYTICAL SOLUTION.
C      K=2, FOR NUMERICAL SOLUTION USING CRANK-NICOLSON METHOD
C           WITH CENTERED-IN-SPACE APPROXIMATION.
C      K=3, FOR NUMERICAL SOLUTION USING CRANK-NICOLSON METHOD
C           WITH BACKWARD-IN-SPACE APPROXIMATION.
```

```
C      K=4, FOR NUMERICAL SOLUTION USING CRANK-NICOLSON METHOD
C           WITH FORWARD-IN-SPACE APPROXIMATION.
C      SET INITIAL CONDITION AT T(1)=0 FOR X(IX)>0.
       DO 30 IX=NBP1, NXP1
30     C(K,1,IX)=0.
       IF (IL.GT.1) GO TO 50
C      SET UPPER BOUNDARY CONDITION AT X(1)=0 FOR T(IT)> OR =0
       DO 40 IT=1, NTP1
40     C(K,IT,1)=C0*DEXP(-G*DT*(IT-1))
       IF ((K.EQ.1).AND.(NL.EQ.1))  CALL ANALY(K,C,NN)
50     CONTINUE
       IF (K.EQ.2)  CALL CNCENT(K,C)
       IF (K.EQ.3)  CALL CNBACK(K,C)
       IF (K.EQ 4)  CALL CNFORW(K,C)
90     CONTINUE
       IF (NN.EQ.1)  PRINT 300,  ((X(IX), T(IT), (C(K,IT,IX), K=1,4),
      $                          IT=1,NTP1,NTOUT), IX=1,NA,NXOUT)
       IF (NN.GT.1)  PRINT 330,  ((X(IX), T(IT), (C(K,IT,IX), K=2,4),
      $                          IT=1,NTP1,NTOUT), IX=1,NA,NXOUT)
       PRINT 350,  ID
       IF (NN.EQ.1)  PRINT 400,  ((T(IT), X(IX), (C(K,IT,IX), K=1,4),
      $                          IX=1,NA,NXOUT), IT=1,NTP1,NTOUT)
       IF (NN.GT.1)  PRINT 440,  ((T(IT), X(IX), (C(K,IT,IX), K=2,4),
      $                          IX=1,NA,NXOUT), IT=1,NTP1,NTOUT)
          GO TO 11111
99999 PRINT 1000
100    FORMAT(1H1,/////,30X,'TEST DATA NO.',I3,//,
      $/,30X,'NUMBER OF LAYERS                         NL =',I5,
      $/,30X,'TOTAL SPACE STEPS OF ALL LAYERS          NX =',I5,
      $/,30X,'NUMBER OF TIME STEPS USED                NT =',I5,
      $/,30X,'SPACE STEPS FOR EACH PRINTOUT         NXOUT =',I5,
      $/,30X,'TIME STEPS FOR EACH PRINTOUT          NTOUT =',I5,
      $/,30X,'SPACE STEP-SIZE                          DX =',F10.4,' (METER)',
      $/,30X,'TIME STEP-SIZE                           DT =',F10.4,' (YEAR)',
      $/,30X,'COMBINED THICKNESS OF ALL LAYERS         XX =',F10.4,' (METER)',
      $/,30X,'TIME PERIOD SIMULATED                    TT =',F10.4,' (YEAR)',
      $/,30X,'INITIAL CONCENTRATION AT X=0             C0 =',F10.4,
      $       ' (MASS/VOLUME)',
      $/,30X,'DECAY COEFFICIENT FOR THE SOURCE          G =',F10.4,' (1/YEAR)')
200    FORMAT(//,30X,'LAYER',I3,',',
      $/,30X,'EFFECTIVE DISPERSION COEFFICIENT          D =',F10.4,
      $       ' (SQUARE METER/YEAR)',
      $/,30X,'EFFECTIVE PORE-WATER VELOCITY             V =',F10.4,
      $       ' (METER/YEAR)',
      $/,30X,'EFFECTIVE TRANSFORMATION COEFF.           P =',F10.4,
      $       ' (1/YEAR)',
      $/,30X,'THICKNESS OF THIS LAYER               THICK =',F10.4,
      $       ' (METER)',
      $/,30X,'DISTANCE SIMULATED FOR THIS LAYER        XL =',F10.4,
      $       ' (METER)')
300    FORMAT(1H1,/////,52X,'ANALYTICAL',6X,'CRANK-NICOLSON METHOD WITH',
      $       /,29X,'DISTANCE',6X,'TIME',6X,'SOLUTION',4X,'CENTERED',
      $       4X,'BACKWARD',4X,'FORWARD',//,(24X,6F12.4))
330    FORMAT(1H1,/////,56X,'CRANK-NICOLSON METHOD WITH',
      $       /,29X,'DISTANCE',6X,'TIME',6X,'CENTERED',4X,'BACKWARD',
      $       4X,'FORWARD',//,(24X,5F12.4))
350    FORMAT(1H1,/////,30X,'TEST DATA NO.',I3,' (CONTINUED)')
400    FORMAT(//,52X,'ANALYTICAL',6X,'CRANK-NICOLSON METHOD WITH',
      $       /,31X,'TIME',6X,'DISTANCE',4X,'SOLUTION',4X,'CENTERED',
      $       4X,'BACKWARD',4X,'FORWARD',//,(24X,6F12.4))
```

```
  440 FORMAT(//,56X,'CRANK-NICOLSON METHOD WITH',/,31X,'TIME',
     $       6X,'DISTANCE',4X,'CENTERED',4X,'BACKWARD',4X,'FORWARD',
     $       //,(24X,5F12.4))
 1000 FORMAT(1H1)
      STOP
      END
C
C
C
C
C     SUBROUTINE "ANALY":
C     FOR ANALYTICAL SOLUTION

      SUBROUTINE ANALY(K,C,NN)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON D,V,P,G,DX,DT,NX,NT,NXP1,NTP1,NA,NB,NBP1,NBP2
      DIMENSION  C(4,201,501)
      H1=V*V+4.*D*(P-G)
      IF ((D.LE O.).OR.(H1.LT.O.)) NN=2
      IF ((D.LE.O.).OR.(H1.LT O.)) RETURN
      H=DSQRT(H1)
      DO 10 IT=2, NTP1
      T=DT*(IT-1)
      DO 10 IX=NBP1, NA
      X=DX*(IX-1)
      R1=(V-H)*X/(2.*D)
      R2=(V+H)*X/(2.*D)
      S1=(X-H*T)/(2.*DSQRT(D*T))
      S2=(X+H*T)/(2.*DSQRT(D*T))
      C(K,IT,IX)=C(K,IT,1)*0.5*(EXF(R1,S1)+EXF(R2,S2))
   10 CONTINUE
      RETURN
      END
C
C
C
C
C     FUNCTION "EXF(A,B)":
C     THIS FUNCTION WILL CALCULATE THE PRODUCT OF EXP(A) AND
C     ERFC(B), THAT IS, EXF(A,B)= EXP(A)*ERFC(B).

      DOUBLE PRECISION FUNCTION EXF(A,B)
      IMPLICIT REAL*8 (A-H,O-Z)
      EXF=O.
      IF ((DABS(A).GT.170.).AND.(B.LE.O.)) RETURN
      IF (B.NE.O.) GO TO 10
      EXF=DEXP(A)
          RETURN
   10 E=A-B*B
      IF ((DABS(E).GT.170.).AND.(B.GT.O.)) RETURN
      IF (E.LT.-170.) GO TO 40
      W=DABS(B)
      IF (W.GT.3.) GO TO 20
      U=1./(1.+0.3275911*W)
      Y=U*(0.2548296-U*(0.2844967-U*(1.421414-U*(1.453152-1.061405*U))))
          GO TO 30
   20 Y=0.5641896/(W+0.5/(W+1./(W+1.5/(W+2./(W+2.5/(W+1.))))))
   30 EXF=Y*DEXP(E)
   40 IF (B.LT.O.) EXF=2.*DEXP(A)-EXF
      RETURN
```

```
      END
C
C-----------------------------------------------------------------------
C
C
C     SUBROUTINE "CNCENT":
C     FOR NUMERICAL SOLUTION USING CRANK-NICOLSON METHOD WITH
C     CENTERED-IN-SPACE APPROXIMATION.

      SUBROUTINE CNCENT(K,C)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON D,V,P,G,DX,DT,NX,NT,NXP1,NTP1,NA,NB,NBP1,NBP2
      DIMENSION  C(4,201,501)
C     COMPUTE THE LEFT-HAND-SIDE TRIDIAGONAL ELEMENTS.
      A1=(-2.*D-V*DX)*DT
      A2=4.*DX*DX+(4.*D+2.*DX*DX*P)*DT
      A3=(-2.*D+V*DX)*DT
C     COMPUTE THE RIGHT-HAND-SIDE COEFFICIENTS.
      B1=-A1
      B2=-A2+8.*DX*DX
      B3=-A3
C     SOLVE THE SYSTEM OF EQUATIONS BY CALLING "SOLVE" SUBROUTINE.
      CALL SOLVE(K,A1,A2,A3,B1,B2,B3,C)
      RETURN
      END
C
C-----------------------------------------------------------------------
C
C
C     SUBROUTINE "CNBACK":
C     FOR NUMERICAL SOLUTION USING CRANK-NICOLSON METHOD WITH
C     BACKWARD-IN-SPACE APPROXIMATION.

      SUBROUTINE CNBACK(K,C)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON D,V,P,G,DX,DT,NX,NT,NXP1,NTP1,NA,NB,NBP1,NBP2
      DIMENSION  C(4,201,501)
C     COMPUTE THE LEFT-HAND-SIDE TRIDIAGONAL ELEMENTS.
      A1=(-D-V*DX)*DT
      A2=2.*DX*DX+(2.*D+V*DX+DX*DX*P)*DT
      A3=-D*DT
C     COMPUTE THE RIGHT-HAND-SIDE COEFFICIENTS.
      B1=-A1
      B2=-A2+4.*DX*DX
      B3=-A3
C     SOLVE THE SYSTEM OF EQUATIONS BY CALLING "SOLVE" SUBROUTINE.
      CALL SOLVE(K,A1,A2,A3,B1,B2,B3,C)
      RETURN
      END
C
C-----------------------------------------------------------------------
C
C
C     SUBROUTINE "CNFORW":
C     FOR NUMERICAL SOLUTION USING CRANK-NICOLSON METHOD WITH
C     FORWARD-IN-SPACE APPROXIMATION.

      SUBROUTINE CNFORW(K,C)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON D,V,P,G,DX,DT,NX,NT,NXP1,NTP1,NA,NB,NBP1,NBP2
```

```
      DIMENSION  C(4,201,501)
C     COMPUTE THE LEFT-HAND-SIDE TRIDIAGONAL ELEMENTS
      A1=-D*DT
      A2=2.*DX*DX+(2.*D-V*DX+DX*DX*P)*DT
      A3=(-D+V*DX)*DT
C     COMPUTE THE RIGHT-HAND-SIDE COEFFICIENTS
      B1=-A1
      B2=-A2+4 *DX*DX
      B3=-A3
C     SOLVE THE SYSTEM OF EQUATIONS BY CALLING "SOLVE" SUBROUTINE.
      CALL SOLVE(K,A1,A2,A3,B1,B2,B3,C)
      RETURN
      END
C
C_____________________________________________________________________
C
C
C     SUBROUTINE "SOLVE":
C     THIS SUBROUTINE WILL COMPUTE THE RIGHT HAND SIDE AND THEN
C     SOLVE THE TRIDIAGONAL SYSTEM OF EQUATIONS BY CALLING
C     "GAUSS" SUBROUTINE.

      SUBROUTINE SOLVE(K,A1,A2,A3,B1,B2,B3,C)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON D,V,P,G,DX,DT,NX,NT,NXP1,NTP1,NA,NB,NBP1,NBP2
      DIMENSION  B(501), CC(501), C(4,201,501)
      DO 40 IT=1, NT
      DO 10 IX=NBP2, NX
10    B(IX)=B1*C(K,IT,IX-1)+B2*C(K,IT,IX)+B3*C(K,IT,IX+1)
      B(NBP1)=B1*C(K,IT,NB)+B2*C(K,IT,NBP1)+B3*C(K,IT,NBP2)
     $        -A1*C(K,IT+1,NB)
C     SET LOWER BOUNDARY CONDITION AT X(NXP1)=XL.
C     ASSUME B3*C(K,IT,NXP1+1)=B3*C(K,IT,NXP1)
C     ASSUME A3*C(K,IT+1,NXP1+1)=A3*C(K,IT,NXP1)
      B(NXP1)=B1*C(K,IT,NX)+(B2+B3-A3)*C(K,IT,NXP1)
C     SOLVE THE TRIDIAGONAL SYSTEM OF EQUATIONS BY CALLING "GAUSS"
C     SUBROUTINE.
      CALL GAUSS(A1,A2,A3,B,CC)
      DO 30 IX=NBP1, NXP1
30    C(K,IT+1,IX)=CC(IX)
40    CONTINUE
      RETURN
      END
C
C_____________________________________________________________________
C
C
C     SUBROUTINE "GAUSS":
C     THIS SUBROUTINE WILL SOLVE A TRIDIAGONAL SYSTEM
C     OF EQUATIONS BASED ON GAUSS ELIMINATION METHOD.
C     THE SYSTEM OF EQUATIONS IS A*C=B.
C     A = TRIDIAGONAL MATRIX WITH NON-ZERO ELEMENTS A1, A2, A3
C     B = KNOWN VALUES ON THE RIGHT HAND SIDE
C     C = UNKNOWNS TO BE SOLVED

      SUBROUTINE GAUSS(A1,A2,A3,B,C)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON D,V,P,G,DX,DT,NX,NT,NXP1,NTP1,NA,NB,NBP1,NBP2
      DIMENSION  B(501), C(501), AA2(501)
C     AA2 IS USED TO AVOID DESTROYING A2 SINCE A2 IS NEEDED FOR REUSE.
```

```
        DO 10 IX=NBP1, NXP1
 10     AA2(IX)=A2
 C      FIRST: FORWARD ELIMINATION (ONLY KEEP NON-ZERO ELEMENTS A2, A3)
        DO 20 IX=NBP2, NXP1
        R=A1/AA2(IX-1)
        AA2(IX)=AA2(IX)-R*A3
 20     B(IX)=B(IX)-R*B(IX-1)
 C      SECOND: BACKWARD SUBSTITUTION (SOLVE FOR UNKNOWNS C).
        C(NXP1)=B(NXP1)/AA2(NXP1)
        NXM1=NXP1-NBP1
        DO 30 J=1, NXM1
        IX=NXP1-J
 30     C(IX)=(B(IX)-A3*C(IX+1))/AA2(IX)
 C      NOTE: A2 IS UNTOUCHED AND BECAUSE OF THE EXISTING OF 'ZERO'
 C             ELEMENTS OUTSIDE THE TRIDIAGONAL AREA, A1 AND A3 ARE
 C             STILL THE SAME AS BEFORE.
        RETURN
        END
 C
 C______________________________________________________________________

 $ENTRY
```

APPENDIX C

SEQUENCE AND FORMAT OF INPUT DATA

The WATFIV compiler has the full FORTRAN IV capabilities. However, since the READ statement is written in free format, which is only available in WATFIV, the model must be modified, with little effort, before it can be applied to any other FORTRAN compilers. The advantages of free format are that there is no FORMAT statement required to refer to the READ statement and the values of input data can be written easily. Each value of the input data is separated by one or more blanks, a comma, or a card. The rules governing the data placement for free format input are:

1. Each time a READ statement is executed, another card is read from the input card deck.

2. If all values punched in a data card are read and the list of variables is not satisfied, additional data cards will be read until the list of variables has been satisfied.

3. If the list is satisfied and there are more values punched in the current data card, they are ignored.

4. Blank cards are ignored.

The first data card of each problem contents the values of NL, NX, NT, NXOUT, NTOUT, XX, TT, CO, and G. The second to the $(NL+1)_{th}$ data cards each contents the values of NSSL, NX, D, V, and P. The notations of the variables are described in the computer program listing (see Appendix B).

The type of a variable corresponds to the type of data the variable represents. An integer variable represents integer data and a real variable represents real data. The type of a variable, such as real or integer, is determined by the predefined specification contained in the FORTRAN language. If the first character of the variable name is I, J, K, L, M, or N, the variable is of the integer type. Otherwise, the variable is of the real type. The variables of the real type are all specified in double precision.

APPENDIX D

SAMPLE OUTPUT

TEST DATA NO 1

```
NUMBER OF LAYERS                          NL =     1
TOTAL SPACE STEPS OF ALL LAYERS           NX =   200
NUMBER OF TIME STEPS USED                 NT =   100
SPACE STEPS FOR EACH PRINTOUT          NXOUT =    10
TIME STEPS FOR EACH PRINTOUT           NTOUT =     5
SPACE STEP-SIZE                           DX =     0 1000 (METER)
TIME STEP-SIZE                            DT =     1.0000 (YEAR)
COMBINED THICKNESS OF ALL LAYERS          XX =    20 0000 (METER)
TIME PERIOD SIMULATED                     TT =   100.0000 (YEAR)
INITIAL CONCENTRATION AT X=O              CO =     1 0000 (MASS/VOLUME)
DECAY COEFFICIENT FOR THE SOURCE           G =     0.0500 (1/YEAR)


LAYER  1,
EFFECTIVE DISPERSION COEFFICIENT           D =     1.0000 (SQUARE METER/YEAR)
EFFECTIVE PORE-WATER VELOCITY              V =     1.0000 (METER/YEAR)
EFFECTIVE TRANSFORMATION COEFF             P =     0 1000 (1/YEAR)
THICKNESS OF THIS LAYER                THICK =    20.0000 (METER)
DISTANCE SIMULATED FOR THIS LAYER         XL =    30.0000 (METER)
```

DISTANCE	TIME	ANALYTICAL SOLUTION	CRANK-NICOLSON METHOD WITH CENTERED	BACKWARD	FORWARD
0.0000	0 0000	1.0000	1.0000	1.0000	1.0000
0.0000	5.0000	0 7788	0 7788	0 7788	0.7788
0.0000	10.0000	0.6065	0.6065	0 6065	0.6065
0.0000	15.0000	0.4724	0.4724	0 4724	0 4724
0.0000	20.0000	0.3679	0.3679	0 3679	0.3679
0 0000	25.0000	0.2865	0 2865	0.2865	0 2865
0.0000	30.0000	0.2231	0.2231	0 2231	0.2231
0.0000	35.0000	0.1738	0 1738	0.1738	0.1738
0.0000	40.0000	0.1353	0.1353	0.1353	0.1353
0.0000	45.0000	0.1054	0 1054	0.1054	0.1054
0.0000	50.0000	0.0821	0.0821	0 0821	0 0821
0.0000	55.0000	0.0639	0.0639	0.0639	0.0639
0.0000	60.0000	0.0498	0.0498	0.0498	0.0498
0.0000	65.0000	0.0388	0.0388	0.0388	0.0388
0.0000	70.0000	0.0302	0.0302	0.0302	0.0302
0.0000	75.0000	0.0235	0.0235	0.0235	0.0235
0.0000	80.0000	0.0183	0.0183	0.0183	0.0183
0.0000	85.0000	0.0143	0.0143	0.0143	0.0143
0.0000	90.0000	0.0111	0.0111	0.0111	0.0111
0.0000	95.0000	0.0087	0.0087	0.0087	0.0087
0.0000	100.0000	0.0067	0.0067	0 0067	0.0067
1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1.0000	5.0000	0.7293	0.6707	0.6678	0.6740
1.0000	10.0000	0.5772	0.5832	0.5858	0.5805
1.0000	15.0000	0.4502	0.4590	0.4581	0.4599
1.0000	20.0000	0.3507	0.3412	0.3411	0.3413
1.0000	25.0000	0.2732	0.2800	0.2806	0.2793
1.0000	30.0000	0.2127	0.2088	0.2081	0.2095
1.0000	35.0000	0.1657	0.1674	0.1680	0.1668
1.0000	40.0000	0.1290	0.1288	0.1283	0.1292
1.0000	45.0000	0.1005	0.0999	0.1002	0.0996
1.0000	50.0000	0.0783	0.0793	0.0791	0.0794
1.0000	55.0000	0.0609	0.0598	0.0599	0.0598
1.0000	60.0000	0.0475	0.0486	0.0486	0.0485
1.0000	65.0000	0.0370	0.0360	0.0359	0.0361
1.0000	70.0000	0.0288	0.0296	0.0297	0.0295
1.0000	75.0000	0.0224	0.0218	0.0217	0.0219
1.0000	80.0000	0.0175	0.0179	0.0181	0.0178
1.0000	85.0000	0.0136	0.0133	0.0132	0.0134
1.0000	90.0000	0.0106	0.0108	0.0109	0.0107
1.0000	95.0000	0.0082	0.0081	0.0081	0.0082
1.0000	100.0000	0.0064	0.0065	0.0065	0.0064
2.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2.0000	5.0000	0.6690	0.6897	0.6886	0.6907
2.0000	10.0000	0.5481	0.5479	0.5470	0.5488
2.0000	15.0000	0.4290	0.4269	0.4270	0.4269
2.0000	20.0000	0.3344	0.3353	0.3356	0.3350
2.0000	25.0000	0.2604	0.2604	0.2603	0.2604
2.0000	30.0000	0.2028	0.2025	0.2026	0.2025
2.0000	35.0000	0.1580	0.1582	0.1583	0.1582
2.0000	40.0000	0.1230	0.1228	0.1228	0.1229
2.0000	45.0000	0.0958	0.0959	0.0959	0.0958

2.0000	50.0000	0.0746	0.0746	0.0746	0.0746
2.0000	55.0000	0.0581	0.0581	0.0581	0.0581
2.0000	60.0000	0.0453	0.0453	0.0453	0.0453
2.0000	65.0000	0.0352	0.0352	0.0352	0.0352
2.0000	70.0000	0.0274	0.0275	0.0275	0.0275
2.0000	75.0000	0.0214	0.0214	0.0214	0.0214
2.0000	80.0000	0.0166	0.0167	0.0167	0.0166
2.0000	85.0000	0.0130	0.0130	0.0130	0.0130
2.0000	90.0000	0.0101	0.0101	0.0101	0.0101
2.0000	95.0000	0.0079	0.0079	0.0079	0.0079
2.0000	100.0000	0.0061	0.0061	0.0061	0.0061
3.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3.0000	5.0000	0.5950	0.5987	0.5992	0.5984
3.0000	10.0000	0.5185	0.5199	0.5199	0.5200
3.0000	15.0000	0.4086	0.4091	0.4092	0.4091
3.0000	20.0000	0.3187	0.3186	0.3186	0.3185
3.0000	25.0000	0.2483	0.2482	0.2483	0.2481
3.0000	30.0000	0.1934	0.1934	0.1935	0.1934
3.0000	35.0000	0.1506	0.1506	0.1506	0.1505
3.0000	40.0000	0.1173	0.1173	0.1173	0.1172
3.0000	45.0000	0.0913	0.0913	0.0914	0.0913
3.0000	50.0000	0.0711	0.0711	0.0711	0.0711
3.0000	55.0000	0.0554	0.0554	0.0554	0.0554
3.0000	60.0000	0.0431	0.0431	0.0432	0.0431
3.0000	65.0000	0.0336	0.0336	0.0336	0.0336
3.0000	70.0000	0.0262	0.0262	0.0262	0.0262
3.0000	75.0000	0.0204	0.0204	0.0204	0.0204
3.0000	80.0000	0.0159	0.0159	0.0159	0.0159
3.0000	85.0000	0.0124	0.0124	0.0124	0.0124
3.0000	90.0000	0.0096	0.0096	0.0096	0.0096
3.0000	95.0000	0.0075	0.0075	0.0075	0.0075
3.0000	100.0000	0.0058	0.0058	0.0058	0.0058
4.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4.0000	5.0000	0.5076	0.5109	0.5109	0.5109
4.0000	10.0000	0.4876	0.4881	0.4876	0.4887
4.0000	15.0000	0.3887	0.3888	0.3888	0.3887
4.0000	20.0000	0.3038	0.3038	0.3039	0.3037
4.0000	25.0000	0.2367	0.2367	0.2368	0.2366
4.0000	30.0000	0.1844	0.1843	0.1844	0.1843
4.0000	35.0000	0.1436	0.1436	0.1436	0.1435
4.0000	40.0000	0.1118	0.1118	0.1119	0.1118
4.0000	45.0000	0.0871	0.0871	0.0871	0.0870
4.0000	50.0000	0.0678	0.0678	0.0678	0.0678
4.0000	55.0000	0.0528	0.0528	0.0528	0.0528
4.0000	60.0000	0.0411	0.0411	0.0412	0.0411
4.0000	65.0000	0.0320	0.0320	0.0320	0.0320
4.0000	70.0000	0.0249	0.0249	0.0250	0.0249
4.0000	75.0000	0.0194	0.0194	0.0194	0.0194
4.0000	80.0000	0.0151	0.0151	0.0151	0.0151
4.0000	85.0000	0.0118	0.0118	0.0118	0.0118
4.0000	90.0000	0.0092	0.0092	0.0092	0.0092
4.0000	95.0000	0.0071	0.0071	0.0072	0.0071
4.0000	100.0000	0.0056	0.0056	0.0056	0.0056
5.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5.0000	5.0000	0.4112	0.4181	0.4201	0.4161
5.0000	10.0000	0.4546	0.4564	0.4558	0.4571
5.0000	15.0000	0.3693	0.3695	0.3695	0.3696
5.0000	20.0000	0.2894	0.2894	0.2895	0.2893
5.0000	25.0000	0.2256	0.2256	0.2257	0.2255
5.0000	30.0000	0.1758	0.1758	0.1758	0.1757

5.0000	35.0000	0.1369	0.1369	0.1369	0.1368
5.0000	40.0000	0.1066	0.1066	0.1067	0.1065
5.0000	45.0000	0.0830	0.0830	0.0831	0.0830
5.0000	50.0000	0.0647	0.0647	0.0647	0.0646
5.0000	55.0000	0.0504	0.0504	0.0504	0.0503
5.0000	60.0000	0.0392	0.0392	0.0392	0.0392
5.0000	65.0000	0.0305	0.0305	0.0306	0.0305
5.0000	70.0000	0.0238	0.0238	0.0238	0.0238
5.0000	75.0000	0.0185	0.0185	0.0185	0.0185
5.0000	80.0000	0.0144	0.0144	0.0144	0.0144
5.0000	85.0000	0.0112	0.0112	0.0112	0.0112
5.0000	90.0000	0.0088	0.0088	0.0088	0.0087
5.0000	95.0000	0.0068	0.0068	0.0068	0.0068
5.0000	100.0000	0.0053	0.0053	0.0053	0.0053
6.0000	0.0000	0.0000	0.0000	0.0000	0.0000
6.0000	5.0000	0.3135	0.3155	0.3201	0.3105
6.0000	10.0000	0.4189	0.4210	0.4205	0.4216
6.0000	15.0000	0.3500	0.3504	0.3503	0.3505
6.0000	20.0000	0.2756	0.2757	0.2758	0.2756
6.0000	25.0000	0.2151	0.2151	0.2152	0.2150
6.0000	30.0000	0.1676	0.1676	0.1677	0.1675
6.0000	35.0000	0.1305	0.1305	0.1306	0.1304
6.0000	40.0000	0.1016	0.1016	0.1017	0.1016
6.0000	45.0000	0.0792	0.0792	0.0792	0.0791
6.0000	50.0000	0.0616	0.0616	0.0617	0.0616
6.0000	55.0000	0.0480	0.0480	0.0480	0.0480
6.0000	60.0000	0.0374	0.0374	0.0374	0.0374
6.0000	65.0000	0.0291	0.0291	0.0291	0.0291
6.0000	70.0000	0.0227	0.0227	0.0227	0.0227
6.0000	75.0000	0.0177	0.0177	0.0177	0.0177
6.0000	80.0000	0.0138	0.0138	0.0138	0.0137
6.0000	85.0000	0.0107	0.0107	0.0107	0.0107
6.0000	90.0000	0.0083	0.0083	0.0083	0.0083
6.0000	95.0000	0.0065	0.0065	0.0065	0.0065
6.0000	100.0000	0.0051	0.0051	0.0051	0.0051
7.0000	0.0000	0.0000	0.0000	0.0000	0.0000
7.0000	5.0000	0.2232	0.2180	0.2242	0.2114
7.0000	10.0000	0.3802	0.3825	0.3822	0.3829
7.0000	15.0000	0.3307	0.3312	0.3310	0.3314
7.0000	20.0000	0.2623	0.2624	0.2625	0.2624
7.0000	25.0000	0.2050	0.2050	0.2051	0.2049
7.0000	30.0000	0.1597	0.1597	0.1598	0.1596
7.0000	35.0000	0.1244	0.1244	0.1245	0.1243
7.0000	40.0000	0.0969	0.0969	0.0970	0.0968
7.0000	45.0000	0.0755	0.0755	0.0755	0.0754
7.0000	50.0000	0.0588	0.0588	0.0588	0.0587
7.0000	55.0000	0.0458	0.0458	0.0458	0.0457
7.0000	60.0000	0.0356	0.0356	0.0357	0.0356
7.0000	65.0000	0.0278	0.0278	0.0278	0.0277
7.0000	70.0000	0.0216	0.0216	0.0216	0.0216
7.0000	75.0000	0.0168	0.0168	0.0169	0.0168
7.0000	80.0000	0.0131	0.0131	0.0131	0.0131
7.0000	85.0000	0.0102	0.0102	0.0102	0.0102
7.0000	90.0000	0.0080	0.0080	0.0080	0.0079
7.0000	95.0000	0.0062	0.0062	0.0062	0.0062
7.0000	100.0000	0.0048	0.0048	0.0048	0.0048
8.0000	0.0000	0.0000	0.0000	0.0000	0.0000
8.0000	5.0000	0.1474	0.1392	0.1454	0.1329
8.0000	10.0000	0.3385	0.3409	0.3410	0.3409
8.0000	15.0000	0.3109	0.3117	0.3114	0.3120

8.0000	20.0000	0.2494	0.2496	0 2496	0.2495
8.0000	25.0000	0.1953	0.1953	0.1955	0.1952
8.0000	30.0000	0.1523	0.1523	0.1524	0.1522
8.0000	35.0000	0.1186	0.1186	0.1187	0.1185
8.0000	40.0000	0.0924	0.0924	0.0925	0.0923
8.0000	45.0000	0.0720	0.0719	0.0720	0.0719
8.0000	50.0000	0.0560	0.0560	0 0561	0.0560
8.0000	55.0000	0.0436	0.0436	0 0437	0.0436
8.0000	60.0000	0.0340	0.0340	0.0340	0.0340
8.0000	65.0000	0.0265	0.0265	0.0265	0.0264
8.0000	70.0000	0.0206	0.0206	0.0206	0.0206
8.0000	75.0000	0.0161	0.0161	0.0161	0.0160
8.0000	80.0000	0.0125	0.0125	0.0125	0.0125
8.0000	85.0000	0.0097	0.0097	0.0097	0.0097
8.0000	90.0000	0.0076	0.0076	0.0076	0.0076
8.0000	95.0000	0.0059	0.0059	0.0059	0.0059
8.0000	100.0000	0.0046	0.0046	0.0046	0.0046
9.0000	0.0000	0.0000	0.0000	0.0000	0.0000
9.0000	5.0000	0.0900	0.0833	0.0883	0.0781
9.0000	10.0000	0.2947	0.2967	0.2975	0.2960
9.0000	15.0000	0.2906	0.2916	0.2913	0.2919
9.0000	20.0000	0.2368	0.2370	0.2370	0.2370
9.0000	25.0000	0.1861	0.1861	0.1862	0.1860
9.0000	30.0000	0.1452	0.1452	0.1453	0.1450
9.0000	35.0000	0.1131	0.1131	0.1132	0.1130
9.0000	40.0000	0.0881	0.0881	0.0882	0.0880
9.0000	45.0000	0.0686	0.0686	0.0687	0.0685
9.0000	50.0000	0.0534	0.0534	0.0535	0.0534
9.0000	55.0000	0.0416	0.0416	0.0416	0.0416
9.0000	60.0000	0.0324	0.0324	0.0324	0.0324
9.0000	65.0000	0.0252	0.0252	0.0253	0.0252
9.0000	70.0000	0.0197	0.0197	0.0197	0.0196
9.0000	75.0000	0.0153	0.0153	0.0153	0.0153
9.0000	80.0000	0.0119	0.0119	0.0119	0.0119
9.0000	85.0000	0.0093	0.0093	0.0093	0.0093
9.0000	90.0000	0.0072	0.0072	0.0072	0.0072
9.0000	95.0000	0.0056	0.0056	0.0056	0.0056
9.0000	100.0000	0.0044	0.0044	0.0044	0.0044
10.0000	0.0000	0.0000	0.0000	0.0000	0.0000
10.0000	5.0000	0.0505	0.0471	0.0509	0.0435
10.0000	10.0000	0.2500	0.2512	0.2528	0.2496
10.0000	15.0000	0.2696	0.2707	0.2704	0.2710
10.0000	20.0000	0.2244	0.2247	0.2247	0.2247
10.0000	25.0000	0.1772	0.1772	0.1773	0.1771
10.0000	30.0000	0.1384	0.1384	0.1385	0.1382
10.0000	35.0000	0.1078	0.1078	0.1079	0.1077
10.0000	40.0000	0.0840	0.0840	0.0841	0.0839
10.0000	45.0000	0.0654	0.0654	0.0655	0.0653
10.0000	50.0000	0.0509	0.0509	0.0510	0.0509
10.0000	55.0000	0.0397	0.0397	0.0397	0.0396
10.0000	60.0000	0.0309	0.0309	0.0309	0.0309
10.0000	65.0000	0.0241	0.0241	0.0241	0.0240
10.0000	70.0000	0.0187	0.0187	0.0188	0.0187
10.0000	75.0000	0.0146	0.0146	0.0146	0.0146
10.0000	80.0000	0.0114	0.0114	0.0114	0.0114
10.0000	85.0000	0.0089	0.0089	0.0089	0.0088
10.0000	90.0000	0.0069	0.0069	0.0069	0.0069
10.0000	95.0000	0.0054	0.0054	0.0054	0.0054
10.0000	100.0000	0.0042	0.0042	0.0042	0.0042
11.0000	0.0000	0.0000	0.0000	0.0000	0.0000

11.0000	5.0000	0 0260	0.0255	0.028C	0 0231
11.0000	10.0000	0 2060	0.2061	0.2085	0.2036
11.0000	15.0000	0.2477	0.2489	0.2487	0 2491
11.0000	20.0000	0.2121	0 2124	0.2124	0 2125
11.0000	25.0000	0 1686	0 1687	0.1687	0 1686
11.0000	30.0000	0 1319	0.1319	0.1320	0.1317
11.0000	35.0000	0.1028	0.1028	0 1029	0 1027
11.0000	40.0000	0.0801	0.0801	0.0801	0 0800
11.0000	45.0000	0.0624	0.0623	0.0624	0.0623
11.0000	50.0000	0.0486	0.0486	0.0486	0.0485
11.0000	55.0000	0.0378	0.0378	0.0379	0.0378
11.0000	60.0000	0.0295	0.0295	0.0295	0 0294
11 0000	65.0000	0.0229	0.0229	0.0230	0.0229
11.0000	70.0000	0.0179	0.0179	0.0179	0 0178
11.0000	75.0000	0.0139	0.0139	0.0139	0.0139
11.0000	80.0000	0.0108	0.0108	0.0108	0.0108
11.0000	85.0000	0.0084	0.0084	0 0084	0 0084
11.0000	90.0000	0.0066	0 0066	0.0066	0 0066
11.0000	95.0000	0.0051	0 0051	0.0051	0 0051
11.0000	100.0000	0.0040	0.0040	0.0040	0.0040
12.0000	0.0000	0.0000	0.0000	0.0000	0.0000
12.0000	5.0000	0.0123	0.0133	0.0148	0 0118
12.0000	10.0000	0.1644	0.1635	0.1665	0.1602
12.0000	15.0000	0.2249	0.2262	0.2262	0.2262
12.0000	20.0000	0.1997	0.2002	0.2001	0.2003
12.0000	25.0000	0.1603	0.1604	0.1604	0.1603
12.0000	30.0000	0.1256	0.1256	0.1258	0.1255
12.0000	35.0000	0.0980	0.0980	0.0981	0.0979
12.0000	40.0000	0.0763	0.0763	0.0764	0.0762
12.0000	45.0000	0.0594	0.0594	0.0595	0.0594
12.0000	50.0000	0.0463	0.0463	0.0463	0.0462
12.0000	55.0000	0.0361	0.0361	0.0361	0.0360
12.0000	60.0000	0.0281	0.0281	0.0281	0.0280
12.0000	65.0000	0.0219	0.0219	0.0219	0.0218
12.0000	70.0000	0.0170	0.0170	0.0171	0.0170
12.0000	75.0000	0.0133	0.0133	0.0133	0.0132
12.0000	80.0000	0.0103	0.0103	0.0103	0.0103
12.0000	85.0000	0.0080	0.0080	0.0081	0.0080
12.0000	90.0000	0.0063	0.0063	0.0063	0.0063
12.0000	95.0000	0.0049	0.0049	0.0049	0.0049
12.0000	100.0000	0.0038	0.0038	0.0038	0.0038
13.0000	0.0000	0.0000	0.0000	0.0000	0.0000
13.0000	5.0000	0.0053	0.0067	0.0076	0.0059
13.0000	10.0000	0.1268	0.1249	0.1285	0.1213
13.0000	15.0000	0.2016	0.2027	0.2030	0.2024
13.0000	20.0000	0.1872	0.1878	0.1877	0.1879
13.0000	25.0000	0.1522	0.1523	0.1524	0.1522
13.0000	30.0000	0.1197	0.1197	0.1198	0.1196
13.0000	35.0000	0.0934	0.0934	0.0935	0.0933
13.0000	40.0000	0.0728	0.0728	0.0729	0.0727
13.0000	45.0000	0.0567	0.0567	0.0567	0.0566
13.0000	50.0000	0.0441	0.0441	0.0442	0.0441
13.0000	55.0000	0.0344	0.0344	0.0344	0.0343
13.0000	60.0000	0.0268	0.0268	0.0268	0.0267
13.0000	65.0000	0.0209	0.0208	0.0209	0.0208
13.0000	70.0000	0.0162	0.0162	0.0163	0.0162
13.0000	75.0000	0.0126	0.0126	0.0127	0.0126
13.0000	80.0000	0.0098	0.0098	0.0099	0.0098
13.0000	85.0000	0.0077	0.0077	0.0077	0.0077
13.0000	90.0000	0.0060	0.0060	0.0060	0 0060

13.0000	95.0000	0 0047	0.0047	0.0047	0.0046
13.0000	100.0000	0.0036	0.0036	0.0036	0 0036
14.0000	0.0000	0.0000	0.0000	0.0000	0 0000
14.0000	5.0000	0.0021	0.0033	0.0038	0.0028
14 0000	10.0000	0.0942	0.0920	0.0956	0.0882
14.0000	15.0000	0.1779	0.1787	0.1794	0.1781
14.0000	20.0000	0.1746	0.1752	0.1751	0.1753
14.0000	25.0000	0 1442	0.1444	0.1444	0.1444
14.0000	30.0000	0.1139	0.1140	0 1141	0.1138
14.0000	35.0000	0.0890	0.0890	0 0891	0.0889
14.0000	40.0000	0.0694	0.0694	0 0695	0.0693
14.0000	45.0000	0.0540	0.0540	0:0541	0 0539
14.0000	50.0000	0.0421	0.0421	0.0421	0.0420
14.0000	55.0000	0.0328	0.0328	0.0328	0.0327
14.0000	60.0000	0.0255	0.0255	0.0256	0.0255
14.0000	65.0000	0.0199	0.0199	0.0199	0.0198
14.0000	70.0000	0.0155	0.0155	0.0155	0.0155
14.0000	75.0000	0.0121	0.0121	0.0121	0.0120
14.0000	80.0000	0.0094	0.0094	0.0094	0.0094
14.0000	85.0000	0.0073	0.0073	0.0073	0.0073
14.0000	90.0000	0.0057	0.0057	0.0057	0.0057
14.0000	95.0000	0.0044	0.0044	0.0044	0.0044
14.0000	100.0000	0.0035	0.0035	0.0035	0.0034
15.0000	0.0000	0.0000	0.0000	0.0000	0.0000
15.0000	5.0000	0.0007	0.0016	0.0019	0.0013
15.0000	10.0000	0.0674	0.0652	0.0686	0.0617
15.0000	15.0000	0.1543	0.1548	0.1559	0.1537
15.0000	20.0000	0.1617	0.1624	0.1623	0.1624
15.0000	25.0000	0.1364	0.1366	0.1367	0.1366
15.0000	30.0000	0.1084	0.1084	0.1085	0.1083
15.0000	35.0000	0.0848	0.0848	0.0849	0.0847
15.0000	40.0000	0.0661	0.0661	0.0662	0.0660
15.0000	45.0000	0.0515	0.0515	0.0516	0.0514
15.0000	50.0000	0.0401	0.0401	0.0402	0.0401
15.0000	55.0000	0.0312	0.0312	0.0313	0.0312
15.0000	60.0000	0.0243	0.0243	0.0244	0.0243
15.0000	65.0000	0.0190	0.0189	0.0190	0.0189
15.0000	70.0000	0.0148	0.0148	0.0148	0.0147
15.0000	75.0000	0.0115	0.0115	0.0115	0.0115
15.0000	80.0000	0.0090	0.0090	0.0090	0.0089
15.0000	85.0000	0.0070	0.0070	0.0070	0.0070
15.0000	90.0000	0.0054	0.0054	0.0054	0.0054
15.0000	95.0000	0.0042	0.0042	0.0042	0.0042
15.0000	100.0000	0.0033	0.0033	0.0033	0.0033
16.0000	0.0000	0.0000	0.0000	0.0000	0.0000
16.0000	5.0000	0.0002	0.0007	0.0009	0.0006
16.0000	10.0000	0.0463	0.0446	0.0475	0.0416
16.0000	15.0000	0.1314	0.1315	0.1329	0.1299
16.0000	20.0000	0.1486	0.1493	0.1493	0.1493
16.0000	25.0000	0.1287	0.1289	0.1289	0.1289
16.0000	30.0000	0.1030	0.1031	0.1032	0.1030
16.0000	35.0000	0.0808	0.0808	0.0809	0.0807
16.0000	40.0000	0.0630	0.0630	0.0631	0.0629
16.0000	45.0000	0.0491	0.0491	0.0492	0.0490
16.0000	50.0000	0.0383	0.0382	0.0383	0.0382
16.0000	55.0000	0.0298	0.0298	0.0298	0.0297
16.0000	60.0000	0.0232	0.0232	0.0232	0.0232
16.0000	65.0000	0.0181	0.0181	0.0181	0.0180
16.0000	70.0000	0.0141	0.0141	0.0141	0.0140
16.0000	75.0000	0.0110	0.0110	0.0110	0.0109

16.0000	80.0000	0.0085	0.0085	0.0085	0.0085
16.0000	85.0000	0.0066	0.0066	0.0067	0.0066
16.0000	90.0000	0.0052	0.0052	0.0052	0.0052
16.0000	95.0000	0.0040	0.0040	0.0040	0.0040
16.0000	100.0000	0.0031	0.0031	0.0031	0.0031
17.0000	0.0000	0.0000	0.0000	0.0000	0.0000
17.0000	5.0000	0.0001	0.0003	0.0004	0.0003
17.0000	10.0000	0.0305	0.0294	0.0318	0.0270
17.0000	15.0000	0 1096	0.1093	0.1111	0.1074
17.0000	20.0000	0.1353	0.1360	0.1362	0.1358
17.0000	25.0000	0.1209	0.1213	0.1213	0.1213
17.0000	30.0000	0.0979	0.0979	0.0980	0.0979
17.0000	35.0000	0.0770	0.0770	0.0771	0.0769
17.0000	40.0000	0.0601	0.0601	0.0602	0.0600
17.0000	45.0000	0.0468	0.0468	0.0469	0.0467
17.0000	50.0000	0.0365	0.0365	0.0365	0.0364
17.0000	55.0000	0.0284	0.0284	0.0284	0.0283
17.0000	60.0000	0.0221	0.0221	0.0222	0.0221
17.0000	65.0000	0.0172	0.0172	0.0173	0.0172
17.0000	70.0000	0.0134	0.0134	0.0134	0.0134
17.0000	75.0000	0.0104	0.0104	0.0105	0.0104
17.0000	80.0000	0.0081	0.0081	0.0082	0.0081
17.0000	85.0000	0.0063	0.0063	0.0063	0.0063
17.0000	90.0000	0.0049	0.0049	0.0049	0.0049
17.0000	95.0000	0.0038	0.0038	0.0039	0.0038
17.0000	100.0000	0.0030	0.0030	0.0030	0.0030
18.0000	0.0000	0.0000	0.0000	0.0000	0.0000
18.0000	5.0000	0.0000	0.0002	0.0002	0.0001
18.0000	10.0000	0.0193	0.0188	0.0206	0.0170
18.0000	15.0000	0.0894	0.0888	0.0908	0.0866
18.0000	20.0000	0.1220	0.1225	0.1229	0.1222
18.0000	25.0000	0.1132	0.1136	0.1136	0.1136
18.0000	30.0000	0.0928	0.0929	0.0930	0.0929
18.0000	35.0000	0.0733	0.0733	0.0734	0.0732
18.0000	40.0000	0.0573	0.0573	0.0574	0.0572
18.0000	45.0000	0.0446	0.0446	0.0447	0.0445
18.0000	50.0000	0.0348	0.0348	0.0348	0.0347
18.0000	55.0000	0.0271	0.0271	0.0271	0.0270
18.0000	60.0000	0.0211	0.0211	0.0211	0.0210
18.0000	65.0000	0.0164	0.0164	0.0165	0.0164
18.0000	70.0000	0.0128	0.0128	0.0128	0.0128
18.0000	75.0000	0.0100	0.0100	0.0100	0.0099
18.0000	80.0000	0.0078	0.0078	0.0078	0.0077
18.0000	85.0000	0.0060	0.0060	0.0061	0.0060
18.0000	90.0000	0.0047	0.0047	0.0047	0.0047
18.0000	95.0000	0.0037	0.0037	0.0037	0.0037
18.0000	100.0000	0.0029	0.0029	0.0029	0.0028
19.0000	0.0000	0.0000	0.0000	0.0000	0.0000
19.0000	5.0000	0.0000	0.0001	0.0001	0.0001
19.0000	10.0000	0.0117	0.0116	0.0130	0.0103
19.0000	15.0000	0.0713	0.0704	0.0726	0.0681
19.0000	20.0000	0.1087	0.1091	0.1096	0.1086
19.0000	25.0000	0.1055	0.1059	0.1059	0.1059
19.0000	30.0000	0.0879	0.0880	0.0880	0.0879
19.0000	35.0000	0.0697	0.0698	0.0698	0.0697
19.0000	40.0000	0.0546	0.0546	0.0547	0.0545
19.0000	45.0000	0.0425	0.0425	0.0426	0.0425
19.0000	50.0000	0.0331	0.0331	0.0332	0.0331
19.0000	55.0000	0.0258	0.0258	0.0259	0.0258
19.0000	60.0000	0.0201	0.0201	0.0201	0.0201

19.0000	65.0000	0.0157	0.0157	0 0157	0.0156
19.0000	70.0000	0.0122	0.0122	0.0122	0 0122
19.0000	75.0000	0.0095	0.0095	0.0095	0.0095
19.0000	80.0000	0.0074	0.0074	0.0074	0.0074
19.0000	85.0000	0.0058	0.0058	0.0058	0.0057
19.0000	90.0000	0.0045	0.0045	0.0045	0.0045
19.0000	95.0000	0.0035	0.0035	0.0035	0.0035
19.0000	100.0000	0.0027	0.0027	0.0027	0.0027
20.0000	0.0000	0.0000	0.0000	0.0000	0.0000
20.0000	5.0000	0.0000	0.0000	0.0000	0.0000
20.0000	10.0000	0.0067	0.0070	0.0079	0.0061
20.0000	15.0000	0.0554	0.0545	0.0567	0.0522
20.0000	20.0000	0.0956	0.0959	0.0966	0.0951
20.0000	25.0000	0.0977	0 0981	0.0982	0.0980
20.0000	30.0000	0.0830	0.0832	0.0832	0.0831
20.0000	35.0000	0.0663	0.0663	0.0664	0.0663
20.0000	40.0000	0.0520	0.0520	0.0521	0.0519
20.0000	45.0000	0.0406	0.0406	0.0406	0.0405
20.0000	50.0000	0.0316	0.0316	0.0317	0.0315
20.0000	55.0000	0.0246	0.0246	0.0247	0.0246
20.0000	60.0000	0.0192	0.0192	0.0192	0.0191
20.0000	65.0000	0.0149	0.0149	0.0150	0.0149
20.0000	70.0000	0.0116	0.0116	0.0116	0.0116
20.0000	75.0000	0.0091	0.0091	0.0091	0.0090
20.0000	80.0000	0.0071	0.0071	0.0071	0.0070
20.0000	85.0000	0.0055	0.0055	0.0055	0.0055
20.0000	90.0000	0.0043	0.0043	0.0043	0.0043
20.0000	95.0000	0.0033	0.0033	0.0033	0.0033
20.0000	100.0000	0.0026	0.0026	0.0026	0.0026

TEST DATA NO. 1 (CONTINUED)

TIME	DISTANCE	ANALYTICAL SOLUTION	CRANK-NICOLSON METHOD WITH CENTERED	BACKWARD	FORWARD
0.0000	0.0000	1.0000	1.0000	1.0000	1.0000
0 0000	1.0000	0 0000	0.0000	0 0000	0 0000
0 0000	2.0000	0.0000	0.0000	0.0000	0.0000
0.0000	3.0000	0.0000	0.0000	0.0000	0.0000
0.0000	4.0000	0.0000	0.0000	0.0000	0.0000
0.0000	5.0000	0.0000	0.0000	0.0000	0.0000
0.0000	6.0000	0.0000	0.0000	0.0000	0.0000
0.0000	7.0000	0.0000	0.0000	0.0000	0.0000
0.0000	8.0000	0.0000	0.0000	0.0000	0.0000
0.0000	9.0000	0.0000	0.0000	0.0000	0.0000
0.0000	10.0000	0.0000	0.0000	0.0000	0 0000
0.0000	11.0000	0.0000	0.0000	0.0000	0.0000
0.0000	12.0000	0.0000	0.0000	0.0000	0.0000
0.0000	13.0000	0.0000	0.0000	0.0000	0.0000
0.0000	14.0000	0.0000	0.0000	0.0000	0.0000
0.0000	15.0000	0.0000	0.0000	0.0000	0.0000
0.0000	16.0000	0.0000	0.0000	0.0000	0.0000
0.0000	17.0000	0.0000	0.0000	0.0000	0.0000
0.0000	18.0000	0.0000	0.0000	0.0000	0.0000
0.0000	19.0000	0.0000	0.0000	0.0000	0.0000
0.0000	20.0000	0.0000	0.0000	0.0000	0.0000
5.0000	0.0000	0.7788	0.7788	0.7788	0.7788
5.0000	1.0000	0.7293	0.6707	0.6678	0.6740
5.0000	2.0000	0.6690	0.6897	0.6886	0.6907
5.0000	3.0000	0.5950	0.5987	0.5992	0.5984
5.0000	4.0000	0.5076	0.5109	0.5109	0.5109
5.0000	5.0000	0.4112	0.4181	0.4201	0.4161
5.0000	6.0000	0.3135	0.3155	0.3201	0.3105
5.0000	7.0000	0.2232	0.2180	0.2242	0.2114
5.0000	8.0000	0.1474	0.1392	0.1454	0.1329
5.0000	9.0000	0.0900	0.0833	0.0883	0.0781
5.0000	10.0000	0.0505	0.0471	0.0509	0.0435
5.0000	11.0000	0.0260	0.0255	0.0280	0.0231
5.0000	12.0000	0.0123	0.0133	0.0148	0.0118
5.0000	13.0000	0.0053	0.0067	0.0076	0.0059
5.0000	14.0000	0.0021	0.0033	0.0038	0.0028
5.0000	15.0000	0.0007	0.0016	0.0019	0.0013
5.0000	16.0000	0.0002	0.0007	0.0009	0.0006
5.0000	17.0000	0.0001	0.0003	0.0004	0.0003
5.0000	18.0000	0.0000	0.0002	0.0002	0.0001
5.0000	19.0000	0.0000	0.0001	0.0001	0.0001
5.0000	20.0000	0.0000	0.0000	0.0000	0.0000
10.0000	0.0000	0.6065	0.6065	0.6065	0.6065
10.0000	1.0000	0.5772	0.5832	0.5858	0.5805
10.0000	2.0000	0.5481	0.5479	0.5470	0.5488
10.0000	3.0000	0.5185	0.5199	0.5199	0.5200
10.0000	4.0000	0.4876	0.4881	0.4876	0.4887
10.0000	5.0000	0.4546	0.4564	0.4558	0.4571
10.0000	6.0000	0.4189	0.4210	0.4205	0.4216

10.0000	7.0000	0.3802	0.3825	0 3822	0.3829
10.0000	8.0000	0.3385	0.3409	0.3410	0.3409
10.0000	9.0000	0.2947	0.2967	0.2975	0.2960
10.0000	10.0000	0.2500	0.2512	0.2528	0.2496
10.0000	11.0000	0.2060	0.2061	0.2085	0.2036
10.0000	12.0000	0.1644	0.1635	0.1665	0.1602
10.0000	13.0000	0.1268	0.1249	0.1285	0.1213
10.0000	14.0000	0.0942	0.0920	0.0956	0.0882
10.0000	15.0000	0.0674	0.0652	0.0686	0.0617
10.0000	16.0000	0.0463	0.0446	0.0475	0.0416
10.0000	17.0000	0.0305	0.0294	0.0318	0.0270
10.0000	18.0000	0.0193	0.0188	0.0206	0.0170
10.0000	19.0000	0.0117	0.0116	0 0130	0.0103
10.0000	20.0000	0.0067	0.0070	0.0079	0.0061
15.0000	0.0000	0.4724	0.4724	0.4724	0.4724
15.0000	1.0000	0.4502	0.4590	0.4581	0.4599
15.0000	2.0000	0.4290	0.4269	0.4270	0.4269
15.0000	3.0000	0.4086	0.4091	0.4092	0.4091
15.0000	4.0000	0.3887	0.3888	0.3888	0.3887
15.0000	5.0000	0.3693	0.3695	0.3695	0.3696
15.0000	6.0000	0.3500	0.3504	0.3503	0.3505
15.0000	7.0000	0.3307	0.3312	0.3310	0.3314
15.0000	8.0000	0.3109	0.3117	0.3114	0.3120
15.0000	9.0000	0.2906	0.2916	0.2913	0.2919
15.0000	10.0000	0.2696	0.2707	0.2704	0.2710
15.0000	11.0000	0.2477	0.2489	0.2487	0.2491
15.0000	12.0000	0.2249	0.2262	0.2262	0.2262
15.0000	13.0000	0.2016	0.2027	0.2030	0.2024
15.0000	14.0000	0.1779	0.1787	0.1794	0.1781
15.0000	15.0000	0.1543	0.1548	0.1559	0.1537
15.0000	16.0000	0.1314	0.1315	0.1329	0.1299
15.0000	17.0000	0.1096	0.1093	0.1111	0.1074
15.0000	18.0000	0.0894	0.0888	0.0908	0.0866
15.0000	19.0000	0.0713	0.0704	0.0726	0.0681
15.0000	20.0000	0.0554	0.0545	0.0567	0.0522
20.0000	0.0000	0.3679	0.3679	0.3679	0.3679
20.0000	1.0000	0.3507	0.3412	0.3411	0.3413
20.0000	2.0000	0.3344	0.3353	0.3356	0.3350
20.0000	3.0000	0.3187	0.3186	0.3186	0.3185
20.0000	4.0000	0.3038	0.3038	0.3039	0.3037
20.0000	5.0000	0.2894	0.2894	0.2895	0.2893
20.0000	6.0000	0.2756	0.2757	0.2758	0.2756
20.0000	7.0000	0.2623	0.2624	0.2625	0.2624
20.0000	8.0000	0.2494	0.2496	0.2496	0.2495
20.0000	9.0000	0.2368	0.2370	0.2370	0.2370
20.0000	10.0000	0.2244	0.2247	0.2247	0.2247
20.0000	11.0000	0.2121	0.2124	0.2124	0.2125
20.0000	12.0000	0.1997	0.2002	0.2001	0.2003
20.0000	13.0000	0.1872	0.1878	0.1877	0.1879
20.0000	14.0000	0.1746	0.1752	0.1751	0.1753
20.0000	15.0000	0.1617	0.1624	0.1623	0.1624
20.0000	16.0000	0.1486	0.1493	0.1493	0.1493
20.0000	17.0000	0.1353	0.1360	0.1362	0.1358
20.0000	18.0000	0.1220	0.1225	0.1229	0.1222
20.0000	19.0000	0.1087	0.1091	0.1096	0.1086
20.0000	20.0000	0.0956	0.0959	0.0966	0.0951
25.0000	0.0000	0.2865	0.2865	0.2865	0.2865
25.0000	1.0000	0.2732	0.2800	0.2806	0.2793
25.0000	2.0000	0.2604	0.2604	0.2603	0.2604
25.0000	3.0000	0.2483	0.2482	0.2483	0.2481

25 0000	4.0000	0.2367	0.2367	0.2368	0 2366
25.0000	5.0000	0.2256	0.2256	0 2257	0 2255
25.0000	6.0000	0.2151	0.2151	0 2152	0 2150
25.0000	7.0000	0.2050	0.2050	0.2051	0.2049
25.0000	8.0000	0.1953	0.1953	0 1955	0.1952
25.0000	9.0000	0.1861	0.1861	0.1862	0.1860
25.0000	10.0000	0.1772	0.1772	0 1773	0.1771
25.0000	11.0000	0.1686	0.1687	0.1687	0.1686
25.0000	12.0000	0.1603	0.1604	0.1604	0 1603
25.0000	13.0000	0.1522	0.1523	0.1524	0.1522
25.0000	14.0000	0.1442	0.1444	0.1444	0.1444
25.0000	15.0000	0.1364	0 1366	0.1367	0 1366
25.0000	16.0000	0.1287	0.1289	0 1289	0.1289
25.0000	17.0000	0.1209	0.1213	0 1213	0.1213
25.0000	18.0000	0.1132	0 1136	0.1136	0.1136
25.0000	19.0000	0.1055	0.1059	0.1059	0.1059
25.0000	20.0000	0.0977	0.0981	0.0982	0.0980
30.0000	0.0000	0.2231	0.2231	0.2231	0.2231
30.0000	1.0000	0.2127	0.2088	0.2081	0.2095
30.0000	2.0000	0.2028	0.2025	0.2026	0 2025
30.0000	3.0000	0.1934	0.1934	0.1935	0.1934
30.0000	4.0000	0.1844	0.1843	0.1844	0.1843
30.0000	5.0000	0.1758	0.1758	0.1758	0.1757
30.0000	6.0000	0.1676	0.1676	0.1677	0.1675
30.0000	7.0000	0.1597	0.1597	0.1598	0.1596
30.0000	8.0000	0.1523	0.1523	0.1524	0.1522
30.0000	9.0000	0.1452	0.1452	0 1453	0.1450
30.0000	10.0000	0.1384	0.1384	0.1385	0.1382
30.0000	11.0000	0.1319	0.1319	0.1320	0.1317
30.0000	12.0000	0.1256	0.1256	0.1258	0.1255
30.0000	13.0000	0.1197	0.1197	0.1198	0.1196
30.0000	14.0000	0.1139	0.1140	0.1141	0.1138
30.0000	15.0000	0.1084	0.1084	0.1085	0.1083
30.0000	16.0000	0.1030	0.1031	0.1032	0.1030
30.0000	17.0000	0.0979	0.0979	0.0980	0.0979
30.0000	18.0000	0.0928	0.0929	0.0930	0.0929
30.0000	19.0000	0.0879	0.0880	0.0880	0.0879
30.0000	20.0000	0.0830	0.0832	0.0832	0.0831
35.0000	0.0000	0.1738	0.1738	0.1738	0.1738
35.0000	1.0000	0.1657	0.1674	0.1680	0.1668
35.0000	2.0000	0.1580	0.1582	0.1583	0.1582
35.0000	3.0000	0.1506	0.1506	0.1506	0.1505
35.0000	4.0000	0.1436	0.1436	0.1436	0.1435
35.0000	5.0000	0.1369	0.1369	0.1369	0.1368
35.0000	6.0000	0.1305	0.1305	0.1306	0.1304
35.0000	7.0000	0.1244	0.1244	0.1245	0.1243
35.0000	8.0000	0.1186	0.1186	0.1187	0.1185
35.0000	9.0000	0.1131	0.1131	0.1132	0.1130
35.0000	10.0000	0.1078	0.1078	0.1079	0.1077
35.0000	11.0000	0.1028	0.1028	0.1029	0.1027
35.0000	12.0000	0.0980	0.0980	0.0981	0.0979
35.0000	13.0000	0.0934	0.0934	0.0935	0.0933
35.0000	14.0000	0.0890	0.0890	0.0891	0.0889
35.0000	15.0000	0.0848	0.0848	0.0849	0.0847
35.0000	16.0000	0.0808	0.0808	0.0809	0.0807
35.0000	17.0000	0.0770	0.0770	0.0771	0.0769
35.0000	18.0000	0.0733	0.0733	0.0734	0.0732
35.0000	19.0000	0.0697	0.0698	0.0698	0.0697
35.0000	20.0000	0.0663	0.0663	0.0664	0.0663
40 0000	0.0000	0.1353	0.1353	0.1353	0.1353

40.0000	1.0000	0.1290	0.1288	0.1283	0.1292
40.0000	2.0000	0.1230	0.1228	0.1228	0.1229
40.0000	3.0000	0.1173	0.1173	0.1173	0.1172
40.0000	4.0000	0.1118	0.1118	0.1119	0.1118
40.0000	5.0000	0.1066	0.1066	0.1067	0.1065
40.0000	6.0000	0.1016	0.1016	0.1017	0 1016
40.0000	7.0000	0.0969	0.0969	0.0970	0.0968
40.0000	8.0000	0.0924	0.0924	0.0925	0.0923
40.0000	9.0000	0.0881	0.0881	0.0882	0.0880
40.0000	10.0000	0.0840	0.0840	0.0841	0.0839
40.0000	11.0000	0.0801	0.0801	0.0801	0.0800
40.0000	12.0000	0.0763	0.0763	0.0764	0.0762
40.0000	13.0000	0.0728	0.0728	0.0729	0.0727
40.0000	14.0000	0.0694	0.0694	0.0695	0 0693
40.0000	15.0000	0.0661	0.0661	0.0662	0 0660
40.0000	16.0000	0.0630	0.0630	0.0631	0.0629
40.0000	17.0000	0.0601	0.0601	0.0602	0.0600
40.0000	18.0000	0.0573	0.0573	0.0574	0.0572
40.0000	19.0000	0.0546	0.0546	0.0547	0.0545
40.0000	20.0000	0.0520	0.0520	0.0521	0.0519
45.0000	0.0000	0.1054	0.1054	0.1054	0.1054
45.0000	1.0000	0.1005	0.0999	0.1002	0.0996
45.0000	2.0000	0.0958	0.0959	0.0959	0.0958
45.0000	3.0000	0.0913	0.0913	0.0914	0.0913
45.0000	4.0000	0.0871	0.0871	0.0871	0.0870
45.0000	5.0000	0.0830	0.0830	0.0831	0.0830
45.0000	6.0000	0.0792	0.0792	0.0792	0.0791
45.0000	7.0000	0.0755	0.0755	0.0755	0.0754
45.0000	8.0000	0.0720	0.0719	0.0720	0.0719
45.0000	9.0000	0.0686	0.0686	0.0687	0.0685
45.0000	10.0000	0.0654	0.0654	0.0655	0.0653
45.0000	11.0000	0.0624	0.0623	0.0624	0.0623
45.0000	12.0000	0.0594	0.0594	0.0595	0.0594
45.0000	13.0000	0.0567	0.0567	0.0567	0.0566
45.0000	14.0000	0.0540	0.0540	0.0541	0.0539
45.0000	15.0000	0.0515	0.0515	0.0516	0.0514
45.0000	16.0000	0.0491	0.0491	0.0492	0.0490
45.0000	17.0000	0.0468	0.0468	0.0469	0.0467
45.0000	18.0000	0.0446	0.0446	0.0447	0.0445
45.0000	19.0000	0.0425	0.0425	0.0426	0.0425
45.0000	20.0000	0.0406	0.0406	0.0406	0.0405
50.0000	0.0000	0.0821	0.0821	0.0821	0.0821
50.0000	1.0000	0.0783	0.0793	0.0791	0.0794
50.0000	2.0000	0.0746	0.0746	0.0746	0.0746
50.0000	3.0000	0.0711	0.0711	0.0711	0.0711
50.0000	4.0000	0.0678	0.0678	0.0678	0.0678
50.0000	5.0000	0.0647	0.0647	0.0647	0.0646
50.0000	6.0000	0.0616	0.0616	0.0617	0.0616
50.0000	7.0000	0.0588	0.0588	0.0588	0.0587
50.0000	8.0000	0.0560	0.0560	0.0561	0.0560
50.0000	9.0000	0.0534	0.0534	0.0535	0.0534
50.0000	10.0000	0.0509	0.0509	0.0510	0.0509
50.0000	11.0000	0.0486	0.0486	0.0486	0.0485
50.0000	12.0000	0.0463	0.0463	0.0463	0.0462
50.0000	13.0000	0.0441	0.0441	0.0442	0.0441
50.0000	14.0000	0.0421	0.0421	0.0421	0.0420
50.0000	15.0000	0.0401	0.0401	0.0402	0.0401
50.0000	16.0000	0.0383	0.0382	0.0383	0.0382
50.0000	17.0000	0.0365	0.0365	0.0365	0.0364
50.0000	18.0000	0.0348	0.0348	0.0348	0.0347

50.0000	19.0000	0.0331	0.0331	0 0332	0 0331
50.0000	20.0000	0.0316	0 0316	0.0317	0.0315
55.0000	0.0000	0.0639	0.0639	0.0639	0.0639
55.0000	1.0000	0.0609	0.0598	0.0599	0.0598
55.0000	2.0000	0.0581	0.0581	0.0581	0 0581
55.0000	3.0000	0.0554	0.0554	0.0554	0.0554
55.0000	4.0000	0.0528	0.0528	0.0528	0.0528
55.0000	5.0000	0.0504	0 0504	0 0504	0 0503
55.0000	6.0000	0.0480	0.0480	0.0480	0.0480
55.0000	7.0000	0.0458	0 0458	0.0458	0.0457
55.0000	8.0000	0.0436	0 0436	0.0437	0.0436
55.0000	9.0000	0.0416	0.0416	0 0416	0.0416
55.0000	10.0000	0.0397	0 0397	0.0397	0.0396
55.0000	11.0000	0.0378	0.0378	0 0379	0.0378
55 0000	12.0000	0.0361	0.0361	0.0361	0.0360
55.0000	13.0000	0.0344	0.0344	0.0344	0.0343
55.0000	14.0000	0.0328	0.0328	0.0328	0 0327
55.0000	15.0000	0.0312	0.0312	0.0313	0.0312
55.0000	16.0000	0.0298	0.0298	0.0298	0.0297
55.0000	17.0000	0.0284	0.0284	0.0284	0.0283
55.0000	18.0000	0.0271	0.0271	0.0271	0.0270
55.0000	19.0000	0.0258	0.0258	0.0259	0 0258
55.0000	20.0000	0.0246	0.0246	0.0247	0.0246
60.0000	0.0000	0.0498	0.0498	0.0498	0.0498
60.0000	1.0000	0.0475	0.0486	0.0486	0.0485
60.0000	2.0000	0.0453	0.0453	0.0453	0.0453
60.0000	3.0000	0.0431	0.0431	0.0432	0.0431
60.0000	4.0000	0.0411	0.0411	0.0412	0.0411
60.0000	5.0000	0.0392	0.0392	0.0392	0.0392
60.0000	6.0000	0.0374	0.0374	0.0374	0.0374
60.0000	7.0000	0.0356	0.0356	0.0357	0.0356
60.0000	8.0000	0.0340	0.0340	0.0340	0.0340
60.0000	9.0000	0.0324	0.0324	0.0324	0.0324
60.0000	10.0000	0.0309	0.0309	0.0309	0.0309
60.0000	11.0000	0.0295	0.0295	0.0295	0.0294
60.0000	12.0000	0.0281	0.0281	0.0281	0.0280
60.0000	13.0000	0.0268	0.0268	0.0268	0.0267
60.0000	14.0000	0.0255	0.0255	0.0256	0.0255
60.0000	15.0000	0.0243	0.0243	0.0244	0.0243
60.0000	16.0000	0.0232	0.0232	0.0232	0.0232
60.0000	17.0000	0.0221	0.0221	0.0222	0.0221
60.0000	18.0000	0.0211	0.0211	0.0211	0.0210
60.0000	19.0000	0.0201	0.0201	0.0201	0.0201
60.0000	20.0000	0.0192	0.0192	0.0192	0.0191
65.0000	0.0000	0.0388	0.0388	0.0388	0.0388
65.0000	1.0000	0.0370	0.0360	0.0359	0.0361
65.0000	2.0000	0.0352	0.0352	0.0352	0.0352
65.0000	3.0000	0.0336	0.0336	0.0336	0.0336
65.0000	4.0000	0.0320	0.0320	0.0320	0.0320
65.0000	5.0000	0.0305	0.0305	0.0306	0.0305
65.0000	6.0000	0.0291	0.0291	0.0291	0.0291
65.0000	7.0000	0.0278	0.0278	0.0278	0.0277
65.0000	8.0000	0.0265	0.0265	0.0265	0.0264
65.0000	9.0000	0.0252	0.0252	0.0253	0.0252
65.0000	10.0000	0.0241	0.0241	0.0241	0.0240
65.0000	11.0000	0.0229	0.0229	0.0230	0.0229
65.0000	12.0000	0.0219	0.0219	0.0219	0.0218
65.0000	13.0000	0.0209	0.0208	0.0209	0.0208
65.0000	14.0000	0.0199	0.0199	0.0199	0.0198
65.0000	15.0000	0.0190	0.0189	0.0190	0 0189

65.0000	16.0000	0 0181	0 0181	0 0181	0 0180
65.0000	17.0000	0.0172	0.0172	0.0173	0 0172
65.0000	18.0000	0.0164	0.0164	0.0165	0.0164
65.0000	19.0000	0.0157	0.0157	0 0157	0.0156
65.0000	20.0000	0.0149	0.0149	0.0150	0 0149
70.0000	0.0000	0.0302	0.0302	0.0302	0.0302
70.0000	1.0000	0.0288	0 0296	0.0297	0.0295
70.0000	2.0000	0.0274	0.0275	0.0275	0.0275
70.0000	3.0000	0.0262	0 0262	0.0262	0.0262
70.0000	4.0000	0.0249	0.0249	0.0250	0.0249
70.0000	5.0000	0.0238	0.0238	0.0238	0.0238
70.0000	6.0000	0.0227	0.0227	0.0227	0.0227
70.0000	7.0000	0.0216	0.0216	0.0216	0.0216
70.0000	8.0000	0.0206	0.0206	0.0206	0.0206
70.0000	9.0000	0.0197	0.0197	0.0197	0.0196
70.0000	10.0000	0.0187	0.0187	0.0188	0.0187
70.0000	11.0000	0.0179	0.0179	0.0179	0.0178
70.0000	12.0000	0.0170	0.0170	0.0171	0.0170
70.0000	13.0000	0.0162	0.0162	0.0163	0.0162
70.0000	14.0000	0.0155	0.0155	0.0155	0.0155
70.0000	15.0000	0.0148	0.0148	0.0148	0.0147
70.0000	16.0000	0.0141	0.0141	0.0141	0.0140
70.0000	17.0000	0.0134	0.0134	0.0134	0.0134
70.0000	18.0000	0.0128	0.0128	0.0128	0.0128
70.0000	19.0000	0.0122	0.0122	0.0122	0.0122
70.0000	20.0000	0.0116	0.0116	0.0116	0.0116
75.0000	0.0000	0.0235	0.0235	0.0235	0.0235
75.0000	1.0000	0.0224	0.0218	0.0217	0.0219
75.0000	2.0000	0.0214	0.0214	0.0214	0.0214
75.0000	3.0000	0.0204	0.0204	0.0204	0.0204
75.0000	4.0000	0.0194	0.0194	0.0194	0.0194
75.0000	5.0000	0.0185	0.0185	0.0185	0.0185
75.0000	6.0000	0.0177	0.0177	0.0177	0.0177
75.0000	7.0000	0.0168	0.0168	0.0169	0.0168
75.0000	8.0000	0.0161	0.0161	0.0161	0.0160
75.0000	9.0000	0.0153	0.0153	0.0153	0.0153
75.0000	10.0000	0.0146	0.0146	0.0146	0.0146
75.0000	11.0000	0.0139	0.0139	0.0139	0.0139
75.0000	12.0000	0.0133	0.0133	0.0133	0.0132
75.0000	13.0000	0.0126	0.0126	0.0127	0.0126
75.0000	14.0000	0.0121	0.0121	0.0121	0.0120
75.0000	15.0000	0.0115	0.0115	0.0115	0.0115
75.0000	16.0000	0.0110	0.0110	0.0110	0.0109
75.0000	17.0000	0.0104	0.0104	0.0105	0.0104
75.0000	18.0000	0.0100	0.0100	0.0100	0.0099
75.0000	19.0000	0.0095	0.0095	0.0095	0.0095
75.0000	20.0000	0.0091	0.0091	0.0091	0.0090
80.0000	0.0000	0.0183	0.0183	0.0183	0.0183
80.0000	1.0000	0.0175	0.0179	0.0181	0.0178
80.0000	2.0000	0.0166	0.0167	0.0167	0.0166
80.0000	3.0000	0.0159	0.0159	0.0159	0.0159
80.0000	4.0000	0.0151	0.0151	0.0151	0.0151
80.0000	5.0000	0.0144	0.0144	0.0144	0.0144
80.0000	6.0000	0.0138	0.0138	0.0138	0.0137
80.0000	7.0000	0.0131	0.0131	0.0131	0.0131
80.0000	8.0000	0.0125	0.0125	0.0125	0.0125
80.0000	9.0000	0.0119	0.0119	0.0119	0.0119
80.0000	10.0000	0.0114	0.0114	0.0114	0.0114
80.0000	11.0000	0.0108	0.0108	0.0108	0.0108
80.0000	12.0000	0.0103	0.0103	0.0103	0.0103

80.0000	13.0000	0.0098	0.0098	0.0099	0.0098
80.0000	14.0000	0.0094	0.0094	0.0094	0.0094
80.0000	15.0000	0.0090	0.0090	0.0090	0.0089
80.0000	16.0000	0.0085	0.0085	0.0085	0.0085
80.0000	17.0000	0.0081	0.0081	0.0082	0.0081
80.0000	18.0000	0.0078	0.0078	0.0078	0.0077
80.0000	19.0000	0.0074	0.0074	0.0074	0.0074
80 0000	20.0000	0.0071	0.0071	0 0071	0.0070
85.0000	0.0000	0.0143	0.0143	0.0143	0.0143
85.0000	1.0000	0.0136	0.0133	0.0132	0 0134
85.0000	2.0000	0.0130	0.0130	0.0130	0.0130
85.0000	3.0000	0.0124	0.0124	0.0124	0.0124
85.0000	4.0000	0.0118	0.0118	0 0118	0.0118
85 0000	5.0000	0.0112	0.0112	0.0112	0.0112
85.0000	6.0000	0.0107	0.0107	0.0107	0.0107
85.0000	7.0000	0.0102	0.0102	0.0102	0.0102
85.0000	8.0000	0.0097	0.0097	0.0097	0.0097
85.0000	9.0000	0.0093	0.0093	0.0093	0.0093
85.0000	10.0000	0.0089	0.0089	0.0089	0.0088
85.0000	11.0000	0.0084	0.0084	0.0084	0.0084
85.0000	12.0000	0.0080	0.0080	0.0081	0.0080
85.0000	13.0000	0.0077	0.0077	0.0077	0.0077
85.0000	14.0000	0.0073	0.0073	0.0073	0.0073
85.0000	15.0000	0.0070	0.0070	0.0070	0.0070
85.0000	16.0000	0.0066	0.0066	0.0067	0.0066
85.0000	17.0000	0.0063	0.0063	0.0063	0.0063
85.0000	18.0000	0.0060	0.0060	0.0061	0.0060
85.0000	19.0000	0.0058	0.0058	0.0058	0.0057
85.0000	20.0000	0.0055	0.0055	0.0055	0.0055
90.0000	0.0000	0.0111	0.0111	0.0111	0.0111
90.0000	1.0000	0.0106	0.0108	0.0109	0.0107
90.0000	2.0000	0.0101	0.0101	0.0101	0.0101
90.0000	3.0000	0.0096	0.0096	0.0096	0.0096
90.0000	4.0000	0.0092	0.0092	0.0092	0.0092
90.0000	5.0000	0.0088	0.0088	0.0088	0.0087
90.0000	6.0000	0.0083	0.0083	0.0083	0.0083
90.0000	7.0000	0.0080	0.0080	0.0080	0.0079
90.0000	8.0000	0.0076	0.0076	0.0076	0.0076
90.0000	9.0000	0.0072	0.0072	0.0072	0.0072
90.0000	10.0000	0.0069	0.0069	0.0069	0.0069
90.0000	11.0000	0.0066	0.0066	0.0066	0.0066
90.0000	12.0000	0.0063	0.0063	0.0063	0.0063
90.0000	13.0000	0.0060	0.0060	0.0060	0.0060
90.0000	14.0000	0.0057	0.0057	0.0057	0.0057
90.0000	15.0000	0.0054	0.0054	0.0054	0.0054
90.0000	16.0000	0.0052	0.0052	0.0052	0.0052
90.0000	17.0000	0.0049	0.0049	0.0049	0.0049
90.0000	18.0000	0.0047	0.0047	0.0047	0.0047
90.0000	19.0000	0.0045	0.0045	0.0045	0.0045
90.0000	20.0000	0.0043	0.0043	0.0043	0.0043
95.0000	0.0000	0.0087	0.0087	0.0087	0.0087
95.0000	1.0000	0.0082	0.0081	0.0081	0.0082
95.0000	2.0000	0.0079	0.0079	0.0079	0.0079
95.0000	3.0000	0.0075	0.0075	0.0075	0.0075
95.0000	4.0000	0.0071	0.0071	0.0072	0.0071
95.0000	5.0000	0.0068	0.0068	0.0068	0.0068
95.0000	6.0000	0.0065	0.0065	0.0065	0.0065
95.0000	7.0000	0.0062	0.0062	0.0062	0.0062
95.0000	8.0000	0.0059	0.0059	0.0059	0.0059
95 0000	9.0000	0.0056	0.0056	0.0056	0.0056

95.0000	10.0000	0.0054	0.0054	0 0054	0.0054
95.0000	11.0000	0.0051	0.0051	0.0051	0.0051
95.0000	12.0000	0.0049	0.0049	0.0049	0.0049
95.0000	13.0000	0.0047	0.0047	0.0047	0.0046
95.0000	14.0000	0.0044	0.0044	0.0044	0.0044
95.0000	15.0000	0.0042	0.0042	0.0042	0.0042
95.0000	16.0000	0.0040	0.0040	0.0040	0.0040
95.0000	17.0000	0.0038	0.0038	0.0039	0.0038
95.0000	18.0000	0.0037	0.0037	0.0037	0.0037
95.0000	19.0000	0.0035	0.0035	0.0035	0.0035
95.0000	20.0000	0.0033	0.0033	0.0033	0.0033
100.0000	0.0000	0.0067	0.0067	0.0067	0.0067
100.0000	1.0000	0.0064	0.0065	0.0065	0.0064
100.0000	2.0000	0.0061	0.0061	0.0061	0.0061
100.0000	3.0000	0.0058	0.0058	0.0058	0.0058
100.0000	4.0000	0.0056	0.0056	0.0056	0.0056
100.0000	5.0000	0.0053	0.0053	0.0053	0.0053
100.0000	6.0000	0.0051	0.0051	0.0051	0.0051
100.0000	7.0000	0.0048	0.0048	0.0048	0.0048
100.0000	8.0000	0.0046	0.0046	0.0046	0.0046
100.0000	9.0000	0.0044	0.0044	0.0044	0.0044
100.0000	10.0000	0.0042	0.0042	0.0042	0.0042
100.0000	11.0000	0.0040	0.0040	0.0040	0.0040
100.0000	12.0000	0.0038	0.0038	0.0038	0.0038
100.0000	13.0000	0.0036	0.0036	0.0036	0.0036
100.0000	14.0000	0.0035	0.0035	0.0035	0.0034
100.0000	15.0000	0.0033	0.0033	0.0033	0.0033
100.0000	16.0000	0.0031	0.0031	0.0031	0.0031
100.0000	17.0000	0.0030	0.0030	0.0030	0 0030
100.0000	18.0000	0.0029	0.0029	0.0029	0.0028
100.0000	19.0000	0.0027	0.0027	0.0027	0.0027
100.0000	20.0000	0.0026	0.0026	0.0026	0.0026

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TEST DATA NO   2

NUMBER OF LAYERS                          NL =    2
TOTAL SPACE STEPS OF ALL LAYERS           NX =  200
NUMBER OF TIME STEPS USED                 NT =  100
SPACE STEPS FOR EACH PRINTOUT          NXOUT =   10
TIME STEPS FOR EACH PRINTOUT           NTOUT =    5
SPACE STEP-SIZE                           DX =    0.1000 (METER)
TIME STEP-SIZE                            DT =    1.0000 (YEAR)
COMBINED THICKNESS OF ALL LAYERS          XX =   20.0000 (METER)
TIME PERIOD SIMULATED                     TT =  100.0000 (YEAR)
INITIAL CONCENTRATION AT X=0              CO =    1.0000 (MASS/VOLUME)
DECAY COEFFICIENT FOR THE SOURCE           G =    0 0500 (1/YEAR)


LAYER  1,
EFFECTIVE DISPERSION COEFFICIENT           D =    0 1000 (SQUARE METER/YEAR)
EFFECTIVE PORE-WATER VELOCITY              V =    0 1000 (METER/YEAR)
EFFECTIVE TRANSFORMATION COEFF.            P =    0.1000 (1/YEAR)
THICKNESS OF THIS LAYER                THICK =    1.0000 (METER)
DISTANCE SIMULATED FOR THIS LAYER         XL =    3.0000 (METER)


LAYER  2,
EFFECTIVE DISPERSION COEFFICIENT           D =    1.0000 (SQUARE METER/YEAR)
EFFECTIVE PORE-WATER VELOCITY              V =    1.0000 (METER/YEAR)
EFFECTIVE TRANSFORMATION COEFF             P =    0.1000 (1/YEAR)
THICKNESS OF THIS LAYER                THICK =   19.0000 (METER)
DISTANCE SIMULATED FOR THIS LAYER         XL =   30.0000 (METER)
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DISTANCE	TIME	CRANK-NICOLSON METHOD WITH CENTERED	BACKWARD	FORWARD
0.0000	0.0000	1.0000	1.0000	1.0000
0.0000	5.0000	0.7788	0.7788	0.7788
0.0000	10.0000	0.6065	0.6065	0 6065
0.0000	15.0000	0.4724	0.4724	0.4724
0.0000	20.0000	0.3679	0.3679	0.3679
0.0000	25.0000	0.2865	0.2865	0.2865
0.0000	30.0000	0.2231	0.2231	0.2231
0.0000	35.0000	0.1738	0.1738	0.1738
0.0000	40.0000	0.1353	0.1353	0.1353
0.0000	45.0000	0.1054	0.1054	0.1054
0.0000	50.0000	0.0821	0.0821	0.0821
0.0000	55.0000	0.0639	0.0639	0.0639
0.0000	60.0000	0.0498	0.0498	0.0498
0.0000	65.0000	0.0388	0.0388	0.0388
0.0000	70.0000	0.0302	0.0302	0.0302
0.0000	75.0000	0.0235	0.0235	0.0235
0.0000	80.0000	0.0183	0.0183	0.0183
0.0000	85.0000	0.0143	0.0143	0.0143
0.0000	90.0000	0.0111	0.0111	0.0111
0.0000	95.0000	0.0087	0.0087	0.0087
0.0000	100.0000	0.0067	0.0067	0.0067
1.0000	0.0000	0.0000	0.0000	0.0000
1.0000	5.0000	0.3371	0.3437	0.3303
1.0000	10.0000	0.3576	0.3605	0.3546
1.0000	15.0000	0.3046	0.3062	0.3029
1.0000	20.0000	0.2462	0.2473	0.2451
1.0000	25.0000	0.1955	0.1964	0.1945
1.0000	30.0000	0.1539	0.1547	0.1532
1.0000	35.0000	0.1207	0.1213	0.1201
1.0000	40.0000	0.0944	0.0949	0.0939
1.0000	45.0000	0.0737	0.0741	0.0733
1.0000	50.0000	0.0575	0.0578	0.0572
1.0000	55.0000	0.0448	0.0451	0.0446
1.0000	60.0000	0.0349	0.0351	0.0347
1.0000	65.0000	0.0272	0.0274	0.0271
1.0000	70.0000	0.0212	0.0213	0.0211
1.0000	75.0000	0.0165	0.0166	0.0164
1.0000	80.0000	0.0129	0.0129	0.0128
1.0000	85.0000	0.0100	0.0101	0.0100
1.0000	90.0000	0.0078	0.0078	0.0078
1.0000	95.0000	0.0061	0.0061	0.0060
1.0000	100.0000	0.0047	0.0048	0.0047
2.0000	0.0000	0.0000	0.0000	0.0000
2.0000	5.0000	0.2745	0.2809	0.2676
2.0000	10.0000	0.3283	0.3312	0.3253
2.0000	15.0000	0.2864	0.2880	0.2848
2.0000	20.0000	0.2334	0.2345	0.2322
2.0000	25.0000	0.1858	0.1867	0.1849
2.0000	30.0000	0.1465	0.1472	0.1458
2.0000	35.0000	0.1149	0.1155	0.1143
2.0000	40.0000	0.0899	0.0904	0.0894
2.0000	45.0000	0.0702	0.0706	0.0698

2.0000	50.0000	0 0548	0 0551	0.0545
2.0000	55.0000	0 0427	0.0430	0.0425
2.0000	60.0000	0 0333	0.0335	0.0331
2.0000	65.0000	0.0259	0.0261	0.0258
2.0000	70.0000	0.0202	0.0203	0.0201
2.0000	75.0000	0.0157	0.0158	0.0157
2.0000	80.0000	0.0123	0.0123	0.0122
2.0000	85.0000	0 0096	0.0096	0.0095
2.0000	90.0000	0.0074	0.0075	0.0074
2.0000	95.0000	0.0058	0.0058	0.0058
2.0000	100.0000	0.0045	0.0045	0.0045
3.0000	0.0000	0.0000	0.0000	0.0000
3.0000	5.0000	0.2122	0.2191	0.2049
3.0000	10.0000	0.2984	0.3015	0.2952
3.0000	15.0000	0.2687	0.2703	0.2670
3.0000	20.0000	0.2209	0.2220	0.2198
3.0000	25.0000	0.1765	0.1773	0.1756
3.0000	30.0000	0.1394	0.1401	0.1387
3.0000	35.0000	0.1094	0.1100	0.1089
3.0000	40.0000	0.0857	0.0861	0.0852
3.0000	45.0000	0.0669	0.0673	0.0666
3.0000	50.0000	0.0522	0.0525	0.0519
3.0000	55.0000	0.0407	0.0410	0.0405
3.0000	60.0000	0.0317	0.0319	0.0316
3.0000	65.0000	0.0247	0.0249	0.0246
3.0000	70.0000	0.0193	0.0194	0.0192
3.0000	75.0000	0.0150	0.0151	0.0149
3.0000	80.0000	0.0117	0.0118	0.0116
3.0000	85.0000	0.0091	0.0092	0.0091
3.0000	90.0000	0.0071	0.0071	0.0070
3.0000	95.0000	0.0055	0.0056	0.0055
3.0000	100.0000	0.0043	0.0043	0.0043
4.0000	0.0000	0.0000	0.0000	0.0000
4.0000	5.0000	0.1522	0.1591	0.1449
4.0000	10.0000	0.2673	0.2705	0.2639
4.0000	15.0000	0.2510	0.2527	0.2493
4.0000	20.0000	0.2089	0.2100	0.2078
4.0000	25.0000	0.1676	0.1684	0.1668
4.0000	30.0000	0.1326	0.1332	0.1319
4.0000	35.0000	0.1042	0.1047	0.1036
4.0000	40.0000	0.0816	0.0820	0.0812
4.0000	45.0000	0.0638	0.0641	0.0634
4.0000	50.0000	0.0498	0.0501	0.0495
4.0000	55.0000	0.0388	0.0390	0.0386
4.0000	60.0000	0.0303	0.0304	0.0301
4.0000	65.0000	0.0236	0.0237	0.0234
4.0000	70.0000	0.0184	0.0185	0.0183
4.0000	75.0000	0.0143	0.0144	0.0142
4.0000	80.0000	0.0111	0.0112	0.0111
4.0000	85.0000	0.0087	0.0087	0.0086
4.0000	90.0000	0.0068	0.0068	0.0067
4.0000	95.0000	0.0053	0.0053	0.0052
4.0000	100.0000	0.0041	0.0041	0.0041
5.0000	0.0000	0.0000	0.0000	0.0000
5.0000	5.0000	0.1010	0.1072	0.0947
5.0000	10.0000	0.2354	0.2388	0.2318
5.0000	15.0000	0.2334	0.2350	0.2316
5.0000	20.0000	0.1972	0.1982	0.1961
5.0000	25.0000	0.1590	0.1598	0.1582
5.0000	30.0000	0.1261	0.1267	0.1254

5.0000	35.0000	0.0992	0.0997	0.0987
5.0000	40.0000	0.0777	0.0781	0 0773
5.0000	45.0000	0.0608	0.0611	0.0604
5.0000	50.0000	0.0474	0.0477	0.0472
5.0000	55.0000	0.0370	0.0372	0.0368
5.0000	60.0000	0.0288	0.0290	0.0287
5.0000	65.0000	0.0225	0.0226	0.0223
5.0000	70.0000	0.0175	0 0176	0.0174
5.0000	75.0000	0.0136	0.0137	0.0136
5.0000	80.0000	0.0106	0.0107	0.0106
5.0000	85.0000	0.0083	0.0083	0.0082
5.0000	90.0000	0.0064	0.0065	0 0064
5.0000	95.0000	0.0050	0 0051	0.0050
5.0000	100.0000	0.0039	0.0039	0 0039
6.0000	0.0000	0.0000	0.0000	0.0000
6.0000	5.0000	0.0626	0.0675	0.0577
6.0000	10.0000	0.2031	0.2068	0 1992
6.0000	15.0000	0.2156	0.2173	0.2138
6.0000	20.0000	0.1858	0.1868	0.1847
6.0000	25.0000	0.1508	0.1516	0.1500
6.0000	30.0000	0.1199	0.1205	0.1192
6.0000	35.0000	0.0944	0.0949	0.0939
6.0000	40.0000	0.0740	0.0744	0.0736
6.0000	45.0000	0.0579	0.0582	0.0576
6.0000	50.0000	0.0452	0.0455	0.0449
6.0000	55.0000	0.0353	0.0355	0.0351
6.0000	60.0000	0.0275	0.0277	0.0273
6.0000	65.0000	0.0214	0.0216	0.0213
6.0000	70.0000	0.0167	0.0168	0.0166
6.0000	75.0000	0.0130	0.0131	0.0129
6.0000	80.0000	0.0101	0.0102	0.0101
6.0000	85.0000	0.0079	0.0079	0.0078
6.0000	90.0000	0.0061	0.0062	0.0061
6.0000	95.0000	0.0048	0.0048	0.0048
6.0000	100.0000	0.0037	0.0038	0.0037
7.0000	0.0000	0.0000	0.0000	0.0000
7.0000	5.0000	0.0365	0.0401	0.0331
7.0000	10.0000	0.1711	0.1750	0.1670
7.0000	15.0000	0.1977	0.1994	0.1958
7.0000	20.0000	0.1745	0.1756	0.1735
7.0000	25.0000	0.1429	0.1436	0.1421
7.0000	30.0000	0.1139	0.1145	0.1133
7.0000	35.0000	0.0898	0.0903	0.0894
7.0000	40.0000	0.0705	0.0709	0.0701
7.0000	45.0000	0.0552	0.0555	0.0548
7.0000	50.0000	0.0431	0.0433	0.0428
7.0000	55.0000	0.0336	0.0338	0.0334
7.0000	60.0000	0.0262	0.0264	0.0260
7.0000	65.0000	0.0204	0.0206	0.0203
7.0000	70.0000	0.0159	0.0160	0.0158
7.0000	75.0000	0.0124	0.0125	0.0123
7.0000	80.0000	0.0097	0.0097	0.0096
7.0000	85.0000	0.0075	0.0076	0.0075
7.0000	90.0000	0.0059	0.0059	0.0058
7.0000	95.0000	0.0046	0.0046	0.0045
7.0000	100.0000	0.0036	0.0036	0.0035
8.0000	0.0000	0.0000	0.0000	0.0000
8.0000	5.0000	0.0203	0.0227	0.0181
8.0000	10.0000	0.1402	0.1443	0.1360
8.0000	15.0000	0.1795	0.1814	0.1776

8.0000	20.0000	0.1635	0.1645	0.1624
8.0000	25.0000	0.1352	0 1359	0.1344
8.0000	30.0000	0.1082	0.1088	0.1076
8.0000	35.0000	0.0855	0.0859	0.0850
8.0000	40.0000	0.0671	0.0675	0.0667
8.0000	45.0000	0.0525	0.0529	0 0522
8.0000	50.0000	0.0411	0.0413	0.0408
8.0000	55.0000	0.0320	0.0322	0 0318
8.0000	60.0000	0.0250	0.0251	0.0248
8.0000	65.0000	0.0195	0.0196	0.0193
8.0000	70.0000	0.0152	0.0153	0 0151
8.0000	75.0000	0.0118	0.0119	0.0117
8 0000	80.0000	0.0092	0.0093	0.0091
8.0000	85.0000	0.0072	0.0072	0.0071
8.0000	90.0000	0.0056	0.0056	0.0055
8.0000	95.0000	0.0044	0.0044	0.0043
8.0000	100.0000	0.0034	0.0034	0.0034
9.0000	0.0000	0.0000	0.0000	0.0000
9.0000	5.0000	0.0108	0.0123	0.0095
9.0000	10.0000	0.1115	0.1156	0 1073
9.0000	15.0000	0.1613	0.1633	0.1592
9.0000	20.0000	0.1525	0.1535	0.1514
9.0000	25.0000	0.1277	0.1284	0.1270
9.0000	30.0000	0.1027	0.1033	0.1021
9.0000	35.0000	0.0813	0.0818	0.0809
9.0000	40.0000	0.0639	0.0643	0.0635
9.0000	45.0000	0.0501	0.0504	0.0498
9.0000	50.0000	0.0391	0.0394	0.0389
9.0000	55.0000	0.0305	0.0307	0.0303
9.0000	60.0000	0.0238	0.0240	0.0237
9.0000	65.0000	0.0186	0.0187	0.0184
9.0000	70.0000	0.0145	0.0146	0.0144
9.0000	75.0000	0.0113	0.0113	0.0112
9.0000	80.0000	0.0088	0.0088	0.0087
9.0000	85.0000	0.0068	0.0069	0.0068
9.0000	90.0000	0.0053	0.0054	0.0053
9.0000	95.0000	0.0041	0.0042	0.0041
9.0000	100.0000	0.0032	0.0033	0.0032
10.0000	0.0000	0.0000	0.0000	0.0000
10.0000	5.0000	0.0056	0.0064	0.0048
10.0000	10.0000	0.0858	0.0897	0.0818
10.0000	15.0000	0.1431	0.1453	0.1409
10.0000	20.0000	0.1416	0.1427	0.1405
10.0000	25.0000	0.1205	0.1212	0.1197
10.0000	30.0000	0.0974	0.0980	0.0969
10.0000	35.0000	0.0773	0.0778	0.0769
10.0000	40.0000	0.0608	0.0612	0.0605
10.0000	45.0000	0.0477	0.0480	0.0474
10.0000	50.0000	0.0373	0.0375	0.0370
10.0000	55.0000	0.0291	0.0293	0.0289
10.0000	60.0000	0.0227	0.0228	0.0226
10.0000	65.0000	0.0177	0.0178	0.0176
10.0000	70.0000	0.0138	0.0139	0.0137
10.0000	75.0000	0.0107	0.0108	0.0107
10.0000	80.0000	0.0084	0.0084	0.0083
10.0000	85.0000	0.0065	0.0066	0.0065
10.0000	90.0000	0.0051	0.0051	0.0050
10.0000	95.0000	0.0040	0.0040	0.0039
10.0000	100.0000	0.0031	0.0031	0.0031
11.0000	0.0000	0.0000	0.0000	0.0000

11.0000	5.0000	0.0028	0.0033	0.0023
11.0000	10.0000	0 0638	0.0674	0.0601
11.0000	15.0000	0.1252	0.1274	0.1228
11.0000	20.0000	0.1307	0.1318	0.1296
11.0000	25.0000	0.1134	0.1140	0.1127
11.0000	30.0000	0.0924	0.0929	0.0918
11.0000	35.0000	0.0735	0.0739	0.0731
11.0000	40.0000	0.0579	0.0583	0.0576
11.0000	45.0000	0.0454	0.0457	0.0451
11.0000	50.0000	0.0355	0.0357	0.0353
11.0000	55.0000	0.0277	0.0279	0.0276
11.0000	60.0000	0.0216	0.0218	0.0215
11.0000	65.0000	0.0169	0.0170	0.0168
11.0000	70.0000	0.0131	0.0132	0.0131
11.0000	75.0000	0.0102	0.0103	0.0102
11.0000	80.0000	0.0080	0.0080	0.0079
11.0000	85.0000	0.0062	0.0063	0.0062
11 0000	90.0000	0.0048	0.0049	0.0048
11.0000	95.0000	0.0038	0.0038	0.0037
11.0000	100.0000	0.0029	0.0030	0.0029
12.0000	0.0000	0.0000	0.0000	0.0000
12.0000	5.0000	0.0014	0.0016	0.0011
12.0000	10.0000	0.0458	0.0490	0.0426
12.0000	15.0000	0.1077	0.1101	0.1052
12.0000	20.0000	0.1199	0.1210	0.1187
12.0000	25.0000	0.1064	0.1071	0.1057
12.0000	30.0000	0.0874	0.0879	0.0869
12.0000	35.0000	0.0698	0.0702	0.0694
12.0000	40.0000	0.0551	0.0554	0.0548
12.0000	45.0000	0.0432	0.0435	0.0430
12.0000	50.0000	0.0338	0.0341	0.0336
12.0000	55.0000	0.0264	0.0266	0.0263
12.0000	60.0000	0.0206	0.0208	0.0205
12.0000	65.0000	0.0161	0.0162	0.0160
12.0000	70.0000	0.0125	0.0126	0.0124
12.0000	75.0000	0.0098	0.0098	0.0097
12.0000	80.0000	0.0076	0.0077	0.0076
12.0000	85.0000	0.0059	0.0060	0.0059
12.0000	90.0000	0.0046	0.0046	0.0046
12.0000	95.0000	0.0036	0.0036	0.0036
12.0000	100.0000	0.0028	0.0028	0.0028
13.0000	0.0000	0.0000	0.0000	0.0000
13.0000	5.0000	0.0006	0.0008	0.0005
13.0000	10.0000	0.0317	0.0344	0.0291
13.0000	15.0000	0.0910	0.0935	0.0885
13.0000	20.0000	0.1091	0.1103	0.1079
13.0000	25.0000	0.0996	0.1003	0.0989
13.0000	30.0000	0.0827	0.0832	0.0822
13.0000	35.0000	0.0663	0.0667	0.0659
13.0000	40.0000	0.0524	0.0527	0.0521
13.0000	45.0000	0.0412	0.0414	0.0409
13.0000	50.0000	0.0322	0.0324	0.0320
13.0000	55.0000	0.0252	0.0253	0.0250
13.0000	60.0000	0.0197	0.0198	0.0195
13.0000	65.0000	0.0153	0.0154	0.0152
13.0000	70.0000	0.0119	0.0120	0.0119
13.0000	75.0000	0.0093	0.0094	0.0092
13.0000	80.0000	0.0073	0.0073	0.0072
13.0000	85.0000	0.0056	0.0057	0.0056
13.0000	90.0000	0.0044	0.0044	0.0044

13.0000	95.0000	0.0034	0.0035	0.0034
13.0000	100.0000	0.0027	0.0027	0.0027
14.0000	0.0000	0.0000	0.0000	0.0000
14.0000	5.0000	0.0003	0.0004	0.0002
14.0000	10.0000	0.0212	0.0233	0.0192
14.0000	15.0000	0.0754	0.0779	0.0729
14.0000	20.0000	0.0984	0.0997	0.0971
14.0000	25.0000	0.0928	0.0935	0.0921
14.0000	30.0000	0.0781	0.0786	0.0776
14 0000	35.0000	0.0629	0.0633	0.0626
14.0000	40.0000	0.0499	0.0502	0.0496
14.0000	45.0000	0.0392	0.0395	0.0390
14 0000	50.0000	0.0307	0.0309	0.0305
14.0000	55.0000	0.0240	0.0242	0.0238
14.0000	60.0000	0.0187	0.0189	0.0186
14.0000	65.0000	0.0146	0.0147	0.0145
14.0000	70.0000	0.0114	0.0115	0.0113
14.0000	75.0000	0.0089	0.0089	0.0088
14.0000	80.0000	0.0069	0.0070	0.0069
14.0000	85.0000	0.0054	0.0054	0.0053
14.0000	90.0000	0.0042	0.0042	0.0042
14.0000	95.0000	0.0033	0.0033	0.0032
14.0000	100.0000	0.0025	0.0026	0.0025
15.0000	0.0000	0.0000	0.0000	0.0000
15.0000	5.0000	0.0001	0.0002	0.0001
15.0000	10.0000	0.0138	0.0154	0.0122
15.0000	15.0000	0.0612	0.0637	0.0587
15.0000	20.0000	0.0879	0.0893	0.0865
15.0000	25.0000	0.0862	0.0869	0.0855
15.0000	30.0000	0.0736	0.0741	0.0731
15.0000	35.0000	0.0597	0.0600	0.0593
15.0000	40.0000	0.0474	0.0477	0.0471
15.0000	45.0000	0.0373	0.0376	0.0371
15.0000	50.0000	0.0292	0.0294	0.0291
15.0000	55.0000	0.0229	0.0230	0.0227
15.0000	60.0000	0.0179	0.0180	0.0177
15.0000	65.0000	0.0139	0.0140	0.0138
15.0000	70.0000	0.0109	0.0109	0.0108
15.0000	75.0000	0.0085	0.0085	0.0084
15.0000	80.0000	0.0066	0.0066	0.0065
15.0000	85.0000	0.0051	0.0052	0.0051
15.0000	90.0000	0.0040	0.0040	0.0040
15.0000	95.0000	0.0031	0.0031	0.0031
15.0000	100.0000	0.0024	0.0024	0.0024
16.0000	0.0000	0.0000	0.0000	0.0000
16.0000	5.0000	0.0001	0.0001	0.0000
16.0000	10.0000	0.0087	0.0098	0.0076
16.0000	15.0000	0.0486	0.0510	0.0462
16.0000	20.0000	0.0777	0.0791	0.0762
16.0000	25.0000	0.0796	0.0803	0.0789
16.0000	30.0000	0.0692	0.0697	0.0688
16.0000	35.0000	0.0566	0.0569	0.0562
16.0000	40.0000	0.0451	0.0453	0.0448
16.0000	45.0000	0.0355	0.0357	0.0353
16.0000	50.0000	0.0279	0.0280	0.0277
16.0000	55.0000	0.0218	0.0219	0.0216
16.0000	60.0000	0.0170	0.0171	0.0169
16.0000	65.0000	0.0133	0.0134	0.0132
16.0000	70.0000	0.0103	0.0104	0.0103
16.0000	75.0000	0.0081	0.0081	0.0080

16.0000	80.0000	0.0063	0.0063	0.0062
16.0000	85.0000	0.0049	0.0049	0.0049
16.0000	90.0000	0.0038	0.0038	0.0038
16.0000	95.0000	0.0030	0.0030	0.0029
16.0000	100.0000	0.0023	0.0023	0.0023
17.0000	0.0000	0.0000	0.0000	0.0000
17.0000	5.0000	0.0000	0.0000	0.0000
17.0000	10.0000	0.0053	0.0061	0.0045
17.0000	15.0000	0.0377	0.0399	0 0355
17.0000	20.0000	0.0678	0.0692	0.0662
17.0000	25.0000	0.0731	0.0738	0.0723
17.0000	30.0000	0.0649	0.0654	0.0645
17.0000	35.0000	0.0535	0.0539	0 0532
17.0000	40.0000	0.0428	0.0431	0.0425
17.0000	45.0000	0.0338	0.0340	0.0336
17.0000	50.0000	0.0265	0.0267	0.0264
17.0000	55.0000	0.0208	0.0209	0.0206
17.0000	60.0000	0.0162	0.0163	0.0161
17.0000	65.0000	0.0126	0.0127	0.0126
17.0000	70.0000	0.0099	0.0099	0.0098
17.0000	75.0000	0.0077	0.0077	0.0076
17.0000	80.0000	0.0060	0.0060	0.0059
17.0000	85.0000	0.0047	0.0047	0.0046
17.0000	90.0000	0.0036	0.0037	0.0036
17.0000	95.0000	0.0028	0.0029	0.0028
17.0000	100.0000	0.0022	0.0022	0.0022
18.0000	0.0000	0.0000	0.0000	0.0000
18.0000	5.0000	0.0000	0.0000	0.0000
18.0000	10.0000	0.0031	0.0037	0.0027
18.0000	15.0000	0.0286	0.0306	0.0266
18.0000	20.0000	0.0584	0.0599	0.0568
18.0000	25.0000	0.0667	0.0675	0.0659
18.0000	30.0000	0.0608	0.0612	0.0603
18.0000	35.0000	0.0506	0.0509	0.0503
18.0000	40.0000	0.0406	0.0409	0.0404
18.0000	45.0000	0.0322	0.0324	0.0319
18.0000	50.0000	0.0253	0.0254	0.0251
18.0000	55.0000	0.0198	0.0199	0.0196
18.0000	60.0000	0.0154	0.0156	0.0153
18.0000	65.0000	0.0121	0.0121	0.0120
18.0000	70.0000	0.0094	0.0095	0.0093
18.0000	75.0000	0.0073	0.0074	0.0073
18.0000	80.0000	0.0057	0.0058	0.0057
18.0000	85.0000	0.0044	0.0045	0.0044
18.0000	90.0000	0.0035	0.0035	0.0034
18.0000	95.0000	0.0027	0.0027	0.0027
18.0000	100.0000	0.0021	0.0021	0.0021
19.0000	0.0000	0.0000	0.0000	0.0000
19.0000	5.0000	0.0000	0.0000	0.0000
19.0000	10.0000	0.0018	0.0022	0.0015
19.0000	15.0000	0.0212	0.0229	0.0195
19.0000	20.0000	0.0495	0.0511	0.0479
19.0000	25.0000	0.0604	0.0612	0.0595
19.0000	30.0000	0.0566	0.0571	0.0562
19.0000	35.0000	0.0478	0.0481	0.0474
19.0000	40.0000	0.0386	0.0388	0.0383
19.0000	45.0000	0.0306	0.0308	0.0304
19.0000	50.0000	0.0240	0.0242	0.0239
19.0000	55.0000	0.0188	0.0190	0.0187
19.0000	60.0000	0.0147	0.0148	0.0146

19.0000	65.0000	0.0115	0.0116	0.0114
19.0000	70.0000	0.0090	0.0090	0.0089
19.0000	75.0000	0.0070	0.0070	0.0069
19.0000	80.0000	0.0054	0.0055	0.0054
19.0000	85.0000	0.0042	0.0043	0.0042
19.0000	90.0000	0.0033	0.0033	0.0033
19.0000	95.0000	0.0026	0.0026	0.0026
19.0000	100.0000	0.0020	0.0020	0.0020
20.0000	0.0000	0.0000	0.0000	0.0000
20.0000	5.0000	0.0000	0.0000	0.0000
20.0000	10.0000	0.0010	0.0013	0.0008
20.0000	15.0000	0.0153	0.0167	0.0139
20.0000	20.0000	0.0414	0.0429	0.0398
20.0000	25.0000	0.0542	0.0551	0.0534
20.0000	30.0000	0.0526	0.0530	0.0521
20.0000	35.0000	0.0450	0.0453	0.0447
20.0000	40.0000	0.0366	0.0368	0.0363
20.0000	45.0000	0.0291	0.0293	0.0289
20.0000	50.0000	0.0229	0.0230	0.0227
20.0000	55.0000	0.0179	0.0181	0.0178
20.0000	60.0000	0.0140	0.0141	0.0139
20.0000	65.0000	0.0110	0.0110	0.0109
20.0000	70.0000	0.0085	0.0086	0.0085
20.0000	75.0000	0.0067	0.0067	0.0066
20.0000	80.0000	0.0052	0.0052	0.0051
20.0000	85.0000	0.0040	0.0041	0.0040
20.0000	90.0000	0.0031	0.0032	0.0031
20.0000	95.0000	0.0025	0.0025	0.0024
20.0000	100.0000	0.0019	0.0019	0.0019

TEST DATA NO 2 (CONTINUED)

TIME	DISTANCE	CRANK-NICOLSON METHOD WITH CENTERED	BACKWARD	FORWARD
0.0000	0.0000	1.0000	1.0000	1.0000
0.0000	1.0000	0.0000	0.0000	0.0000
0.0000	2.0000	0.0000	0.0000	0.0000
0.0000	3.0000	0.0000	0.0000	0.0000
0.0000	4.0000	0.0000	0.0000	0.0000
0.0000	5.0000	0.0000	0.0000	0.0000
0.0000	6.0000	0.0000	0.0000	0.0000
0.0000	7.0000	0.0000	0.0000	0.0000
0.0000	8.0000	0.0000	0.0000	0.0000
0.0000	9.0000	0.0000	0.0000	0.0000
0.0000	10.0000	0.0000	0.0000	0.0000
0.0000	11.0000	0.0000	0.0000	0.0000
0.0000	12.0000	0.0000	0.0000	0.0000
0.0000	13.0000	0.0000	0.0000	0.0000
0.0000	14.0000	0.0000	0.0000	0.0000
0.0000	15.0000	0.0000	0.0000	0.0000
0.0000	16.0000	0.0000	0.0000	0.0000
0.0000	17.0000	0.0000	0.0000	0.0000
0.0000	18.0000	0.0000	0.0000	0.0000
0.0000	19.0000	0.0000	0.0000	0.0000
0.0000	20.0000	0.0000	0.0000	0.0000
5.0000	0.0000	0.7788	0.7788	0.7788
5.0000	1.0000	0.3371	0.3437	0.3303
5.0000	2.0000	0.2745	0.2809	0.2676
5.0000	3.0000	0.2122	0.2191	0.2049
5.0000	4.0000	0.1522	0.1591	0.1449
5.0000	5.0000	0.1010	0.1072	0.0947
5.0000	6.0000	0.0626	0.0675	0.0577
5.0000	7.0000	0.0365	0.0401	0.0331
5.0000	8.0000	0.0203	0.0227	0.0181
5.0000	9.0000	0.0108	0.0123	0.0095
5.0000	10.0000	0.0056	0.0064	0.0048
5.0000	11.0000	0.0028	0.0033	0.0023
5.0000	12.0000	0.0014	0.0016	0.0011
5.0000	13.0000	0.0006	0.0008	0.0005
5.0000	14.0000	0.0003	0.0004	0.0002
5.0000	15.0000	0.0001	0.0002	0.0001
5.0000	16.0000	0.0001	0.0001	0.0000
5.0000	17.0000	0.0000	0.0000	0.0000
5.0000	18.0000	0.0000	0.0000	0.0000
5.0000	19.0000	0.0000	0.0000	0.0000
5.0000	20.0000	0.0000	0.0000	0.0000
10.0000	0.0000	0.6065	0.6065	0.6065
10.0000	1.0000	0.3576	0.3605	0.3546
10.0000	2.0000	0.3283	0.3312	0.3253
10.0000	3.0000	0.2984	0.3015	0.2952
10.0000	4.0000	0.2673	0.2705	0.2639
10.0000	5.0000	0.2354	0.2388	0.2318
10.0000	6.0000	0.2031	0.2068	0.1992

10.0000	7.0000	0.1711	0.1750	0.1670
10.0000	8.0000	0.1402	0.1443	0.1360
10.0000	9.0000	0.1115	0.1156	0.1073
10.0000	10.0000	0.0858	0.0897	0.0818
10.0000	11.0000	0.0638	0.0674	0.0601
10.0000	12.0000	0.0458	0.0490	0.0426
10.0000	13.0000	0.0317	0.0344	0.0291
10.0000	14.0000	0.0212	0.0233	0.0192
10.0000	15.0000	0.0138	0.0154	0.0122
10.0000	16.0000	0.0087	0.0098	0.0076
10.0000	17.0000	0.0053	0.0061	0.0045
10.0000	18.0000	0.0031	0.0037	0.0027
10.0000	19.0000	0.0018	0.0022	0.0015
10.0000	20.0000	0.0010	0.0013	0.0008
15.0000	0.0000	0.4724	0.4724	0.4724
15.0000	1.0000	0.3046	0.3062	0.3029
15.0000	2.0000	0.2864	0.2880	0.2848
15.0000	3.0000	0.2687	0.2703	0.2670
15.0000	4.0000	0.2510	0.2527	0.2493
15.0000	5.0000	0.2334	0.2350	0.2316
15.0000	6.0000	0.2156	0.2173	0.2138
15.0000	7.0000	0.1977	0.1994	0.1958
15.0000	8.0000	0.1795	0.1814	0.1776
15.0000	9.0000	0.1613	0.1633	0.1592
15.0000	10.0000	0.1431	0.1453	0.1409
15.0000	11.0000	0.1252	0.1274	0.1228
15.0000	12.0000	0.1077	0.1101	0.1052
15.0000	13.0000	0.0910	0.0935	0.0885
15.0000	14.0000	0.0754	0.0779	0.0729
15.0000	15.0000	0.0612	0.0637	0.0587
15.0000	16.0000	0.0486	0.0510	0.0462
15.0000	17.0000	0.0377	0.0399	0.0355
15.0000	18.0000	0.0286	0.0306	0.0266
15.0000	19.0000	0.0212	0.0229	0.0195
15.0000	20.0000	0.0153	0.0167	0.0139
20.0000	0.0000	0.3679	0.3679	0.3679
20.0000	1.0000	0.2462	0.2473	0.2451
20.0000	2.0000	0.2334	0.2345	0.2322
20.0000	3.0000	0.2209	0.2220	0.2198
20.0000	4.0000	0.2089	0.2100	0.2078
20.0000	5.0000	0.1972	0.1982	0.1961
20.0000	6.0000	0.1858	0.1868	0.1847
20.0000	7.0000	0.1745	0.1756	0.1735
20.0000	8.0000	0.1635	0.1645	0.1624
20.0000	9.0000	0.1525	0.1535	0.1514
20.0000	10.0000	0.1416	0.1427	0.1405
20.0000	11.0000	0.1307	0.1318	0.1296
20.0000	12.0000	0.1199	0.1210	0.1187
20.0000	13.0000	0.1091	0.1103	0.1079
20.0000	14.0000	0.0984	0.0997	0.0971
20.0000	15.0000	0.0879	0.0893	0.0865
20.0000	16.0000	0.0777	0.0791	0.0762
20.0000	17.0000	0.0678	0.0692	0.0662
20.0000	18.0000	0.0584	0.0599	0.0568
20.0000	19.0000	0.0495	0.0511	0.0479
20.0000	20.0000	0.0414	0.0429	0.0398
25.0000	0.0000	0.2865	0.2865	0.2865
25.0000	1.0000	0.1955	0.1964	0.1945
25.0000	2.0000	0.1858	0.1867	0.1849
25.0000	3.0000	0.1765	0.1773	0.1756

25.0000	4 0000	0.1676	0.1684	0 1668
25.0000	5.0000	0.1590	0.1598	0.1582
25.0000	6.0000	0.1508	0.1516	0.1500
25.0000	7.0000	0.1429	0 1436	0.1421
25.0000	8.0000	0.1352	0.1359	0.1344
25.0000	9.0000	0.1277	0.1284	0.1270
25.0000	10.0000	0.1205	0.1212	0.1197
25.0000	11.0000	0.1134	0.1140	0 1127
25.0000	12.0000	0.1064	0.1071	0.1057
25.0000	13.0000	0.0996	0.1003	0.0989
25.0000	14.0000	0.0928	0.0935	0.0921
25.0000	15.0000	0.0862	0.0869	0.0855
25.0000	16.0000	0.0796	0.0803	0.0789
25.0000	17.0000	0.0731	0.0738	0.0723
25.0000	18.0000	0.0667	0.0675	0.0659
25.0000	19.0000	0.0604	0.0612	0.0595
25.0000	20.0000	0.0542	0.0551	0.0534
30.0000	0.0000	0.2231	0.2231	0.2231
30.0000	1.0000	0.1539	0.1547	0.1532
30.0000	2.0000	0.1465	0.1472	0.1458
30.0000	3.0000	0.1394	0.1401	0.1387
30.0000	4.0000	0.1326	0.1332	0.1319
30.0000	5.0000	0.1261	0.1267	0.1254
30.0000	6.0000	0.1199	0.1205	0.1192
30.0000	7.0000	0.1139	0.1145	0.1133
30.0000	8.0000	0.1082	0.1088	0.1076
30.0000	9.0000	0.1027	0.1033	0.1021
30.0000	10.0000	0.0974	0.0980	0.0969
30.0000	11.0000	0.0924	0.0929	0.0918
30.0000	12.0000	0.0874	0.0879	0.0869
30.0000	13.0000	0.0827	0.0832	0.0822
30.0000	14.0000	0.0781	0.0786	0.0776
30.0000	15.0000	0.0736	0.0741	0.0731
30.0000	16.0000	0.0692	0.0697	0.0688
30.0000	17.0000	0.0649	0.0654	0.0645
30.0000	18.0000	0.0608	0.0612	0.0603
30.0000	19.0000	0.0566	0.0571	0.0562
30.0000	20.0000	0.0526	0.0530	0.0521
35.0000	0.0000	0.1738	0.1738	0.1738
35.0000	1.0000	0.1207	0.1213	0.1201
35.0000	2.0000	0.1149	0.1155	0.1143
35.0000	3.0000	0.1094	0.1100	0.1089
35.0000	4.0000	0.1042	0.1047	0.1036
35.0000	5.0000	0.0992	0.0997	0.0987
35.0000	6.0000	0.0944	0.0949	0.0939
35.0000	7.0000	0.0898	0.0903	0.0894
35.0000	8.0000	0.0855	0.0859	0.0850
35.0000	9.0000	0.0813	0.0818	0.0809
35.0000	10.0000	0.0773	0.0778	0.0769
35.0000	11.0000	0.0735	0.0739	0.0731
35.0000	12.0000	0.0698	0.0702	0.0694
35.0000	13.0000	0.0663	0.0667	0.0659
35.0000	14.0000	0.0629	0.0633	0.0626
35.0000	15.0000	0.0597	0.0600	0.0593
35.0000	16.0000	0.0566	0.0569	0.0562
35.0000	17.0000	0.0535	0.0539	0.0532
35.0000	18.0000	0.0506	0.0509	0.0503
35.0000	19.0000	0.0478	0.0481	0.0474
35.0000	20.0000	0.0450	0.0453	0.0447
40.0000	0.0000	0.1353	0.1353	0.1353

40 0000	1.0000	0.0944	0.0949	0.0939
40.0000	2.0000	0.0899	0.0904	0.0894
40.0000	3.0000	0.0857	0.0861	0.0852
40.0000	4.0000	0.0816	0.0820	0.0812
40.0000	5.0000	0.0777	0.0781	0.0773
40.C000	6.0000	0.0740	0.0744	0.0736
40.0000	7.0000	0.0705	0.0709	0.0701
40.0000	8.0000	0.0671	0.0675	0.0667
40.0000	9.0000	0.0639	0.0643	0.0635
40.0000	10.0000	0.0608	0.0612	0.0605
40.0000	11.0000	0.0579	0.0583	0.0576
40.0000	12.0000	0.0551	0.0554	0.0548
40 0000	13.0000	0.0524	0.0527	0.0521
40.0000	14.0000	0.0499	0.0502	0.0496
40.0000	15.0000	0.0474	0.0477	0.0471
40.0000	16.0000	0.0451	0.0453	0.0448
40.0000	17.0000	0.0428	0.0431	0.0425
40.0000	18.0000	0.0406	0.0409	0.0404
40.0000	19.0000	0.0386	0.0388	0.0383
40.0000	20.0000	0.0366	0.0368	0.0363
45.0000	0.0000	0.1054	0.1054	0.1054
45.0000	1.0000	0.0737	0.0741	0.0733
45.0000	2.0000	0.0702	0.0706	0.0698
45.0000	3.0000	0.0669	0.0673	0:0666
45.0000	4.0000	0.0638	0.0641	0.0634
45.0000	5.0000	0.0608	0.0611	0.0604
45.0000	6.0000	0.0579	0.0582	0.0576
45.0000	7.0000	0.0552	0.0555	0.0548
45.0000	8.0000	0.0525	0.0529	0.0522
45.0000	9.0000	0.0501	0.0504	0.0498
45.0000	10.0000	0.0477	0.0480	0.0474
45.0000	11.0000	0.0454	0.0457	0.0451
45.0000	12.0000	0.0432	0.0435	0.0430
45.0000	13.0000	0.0412	0.0414	0.0409
45.0000	14.0000	0.0392	0.0395	0.0390
45.0000	15.0000	0.0373	0.0376	0.0371
45.0000	16.0000	0.0355	0.0357	0.0353
45.0000	17.0000	0.0338	0.0340	0.0336
45.0000	18.0000	0.0322	0.0324	0.0319
45.0000	19.0000	0.0306	0.0308	0.0304
45.0000	20.0000	0.0291	0.0293	0.0289
50.0000	0.0000	0.0821	0.0821	0.0821
50.0000	1.0000	0.0575	0.0578	0.0572
50.0000	2.0000	0.0548	0.0551	0.0545
50.0000	3.0000	0.0522	0.0525	0.0519
50.0000	4.0000	0.0498	0.0501	0.0495
50.0000	5.0000	0.0474	0.0477	0.0472
50.0000	6.0000	0.0452	0.0455	0.0449
50.0000	7.0000	0.0431	0.0433	0.0428
50.0000	8.0000	0.0411	0.0413	0.0408
50.0000	9.0000	0.0391	0.0394	0.0389
50.0000	10.0000	0.0373	0.0375	0.0370
50.0000	11.0000	0.0355	0.0357	0.0353
50.0000	12.0000	0.0338	0.0341	0.0336
50.0000	13.0000	0.0322	0.0324	0.0320
50.0000	14.0000	0.0307	0.0309	0.0305
50.0000	15.0000	0.0292	0.0294	0.0291
50.0000	16.0000	0.0279	0.0280	0.0277
50.0000	17.0000	0.0265	0.0267	0.0264
50.0000	18.0000	0.0253	0 0254	0.0251

50.0000	19.0000	0.0240	0.0242	0.0239
50.0000	20.0000	0.0229	0.0230	0.0227
55.0000	0.0000	0.0639	0.0639	0.0639
55.0000	1.0000	0.0448	0.0451	0.0446
55.0000	2.0000	0.0427	0.0430	0.0425
55.0000	3.0000	0.0407	0.0410	0.0405
55.0000	4.0000	0.0388	0.0390	0.0386
55.0000	5.0000	0.0370	0.0372	0.0368
55.0000	6.0000	0.0353	0.0355	0.0351
55.0000	7.0000	0.0336	0.0338	0.0334
55.0000	8.0000	0.0320	0.0322	0.0318
55.0000	9.0000	0.0305	0.0307	0.0303
55.0000	10.0000	0.0291	0.0293	0.0289
55.0000	11.0000	0.0277	0.0279	0.0276
55.0000	12.0000	0.0264	0.0266	0.0263
55.0000	13.0000	0.0252	0.0253	0.0250
55.0000	14.0000	0.0240	0.0242	0.0238
55.0000	15.0000	0.0229	0.0230	0.0227
55.0000	16.0000	0.0218	0.0219	0.0216
55.0000	17.0000	0.0208	0.0209	0.0206
55.0000	18.0000	0.0198	0.0199	0.0196
55.0000	19.0000	0.0188	0.0190	0.0187
55.0000	20.0000	0.0179	0.0181	0.0178
60.0000	0.0000	0.0498	0.0498	0.0498
60.0000	1.0000	0.0349	0.0351	0.0347
60.0000	2.0000	0.0333	0.0335	0.0331
60.0000	3.0000	0.0317	0.0319	0.0316
60.0000	4.0000	0.0303	0.0304	0.0301
60.0000	5.0000	0.0288	0.0290	0.0287
60.0000	6.0000	0.0275	0.0277	0.0273
60.0000	7.0000	0.0262	0.0264	0.0260
60.0000	8.0000	0.0250	0.0251	0.0248
60.0000	9.0000	0.0238	0.0240	0.0237
60.0000	10.0000	0.0227	0.0228	0.0226
60.0000	11.0000	0.0216	0.0218	0.0215
60.0000	12.0000	0.0206	0.0208	0.0205
60.0000	13.0000	0.0197	0.0198	0.0195
60.0000	14.0000	0.0187	0.0189	0.0186
60.0000	15.0000	0.0179	0.0180	0.0177
60.0000	16.0000	0.0170	0.0171	0.0169
60.0000	17.0000	0.0162	0.0163	0.0161
60.0000	18.0000	0.0154	0.0156	0.0153
60.0000	19.0000	0.0147	0.0148	0.0146
60.0000	20.0000	0.0140	0.0141	0.0139
65.0000	0.0000	0.0388	0.0388	0.0388
65.0000	1.0000	0.0272	0.0274	0.0271
65.0000	2.0000	0.0259	0.0261	0.0258
65.0000	3.0000	0.0247	0.0249	0.0246
65.0000	4.0000	0.0236	0.0237	0.0234
65.0000	5.0000	0.0225	0.0226	0.0223
65.0000	6.0000	0.0214	0.0216	0.0213
65.0000	7.0000	0.0204	0.0206	0.0203
65.0000	8.0000	0.0195	0.0196	0.0193
65.0000	9.0000	0.0186	0.0187	0.0184
65.0000	10.0000	0.0177	0.0178	0.0176
65.0000	11.0000	0.0169	0.0170	0.0168
65.0000	12.0000	0.0161	0.0162	0.0160
65.0000	13.0000	0.0153	0.0154	0.0152
65.0000	14.0000	0.0146	0.0147	0.0145
65.0000	15.0000	0.0139	0.0140	0.0138

65.0000	16.0000	0.0133	0 0134	0.0132
65.0000	17.0000	0.0126	0.0127	0.0126
65.0000	18.0000	0.0121	0.0121	0.0120
65.0000	19.0000	0.0115	0.0116	0.0114
65.0000	20.0000	0.0110	0.0110	0.0109
70.0000	0.0000	0.0302	0.0302	0.0302
70.0000	1.0000	0.0212	0.0213	0.0211
70.0000	2.0000	0.0202	0.0203	0.0201
70.0000	3.0000	0.0193	0.0194	0.0192
70.0000	4.0000	0.0184	0.0185	0.0183
70.0000	5.0000	0.0175	0.0176	0.0174
70.0000	6.0000	0.0167	0.0168	0.0166
70.0000	7.0000	0.0159	0.0160	0.0158
70.0000	8.0000	0.0152	0.0153	0.0151
70.0000	9.0000	0.0145	0.0146	0 0144
70.0000	10.0000	0.0138	0.0139	0.0137
70.0000	11.0000	0.0131	0.0132	0.0131
70.0000	12.0000	0.0125	0.0126	0.0124
70.0000	13.0000	0.0119	0.0120	0.0119
70.0000	14.0000	0.0114	0 0115	0.0113
70.0000	15.0000	0.0109	0.0109	0.0108
70.0000	16.0000	0.0103	0.0104	0.0103
70.0000	17.0000	0.0099	0.0099	0.0098
70.0000	18.0000	0.0094	0.0095	0.0093
70.0000	19.0000	0.0090	0.0090	0.0089
70.0000	20.0000	0.0085	0.0086	0.0085
75.0000	0.0000	0.0235	0.0235	0.0235
75.0000	1.0000	0.0165	0.0166	0.0164
75.0000	2.0000	0.0157	0.0158	0.0157
75.0000	3.0000	0.0150	0.0151	0.0149
75.0000	4.0000	0.0143	0.0144	0.0142
75.0000	5.0000	0.0136	0.0137	0.0136
75.0000	6.0000	0.0130	0.0131	0.0129
75.0000	7.0000	0.0124	0.0125	0.0123
75.0000	8.0000	0.0118	0.0119	0.0117
75.0000	9.0000	0.0113	0.0113	0.0112
75.0000	10.0000	0.0107	0.0108	0.0107
75.0000	11.0000	0.0102	0.0103	0.0102
75.0000	12.0000	0.0098	0.0098	0.0097
75.0000	13.0000	0.0093	0.0094	0.0092
75.0000	14.0000	0.0089	0.0089	0.0088
75.0000	15.0000	0.0085	0.0085	0.0084
75.0000	16.0000	0.0081	0.0081	0.0080
75.0000	17.0000	0.0077	0.0077	0.0076
75.0000	18.0000	0.0073	0.0074	0.0073
75.0000	19.0000	0.0070	0.0070	0.0069
75.0000	20.0000	0.0067	0.0067	0.0066
80.0000	0.0000	0.0183	0.0183	0.0183
80.0000	1.0000	0.0129	0.0129	0.0128
80.0000	2.0000	0.0123	0.0123	0.0122
80.0000	3.0000	0.0117	0.0118	0.0116
80.0000	4.0000	0.0111	0.0112	0.0111
80.0000	5.0000	0.0106	0.0107	0.0106
80.0000	6.0000	0.0101	0.0102	0.0101
80.0000	7.0000	0.0097	0.0097	0.0096
80.0000	8.0000	0.0092	0.0093	0.0091
80.0000	9.0000	0.0088	0.0088	0.0087
80.0000	10.0000	0.0084	0.0084	0.0083
80.0000	11.0000	0.0080	0.0080	0.0079
80.0000	12.0000	0.0076	0.0077	0 0076

80.0000	13.0000	0 0073	0.0073	0.0072
80.0000	14.0000	0.0069	0.0070	0.0069
80.0000	15.0000	0.0066	0.0066	0.0065
80.0000	16.0000	0.0063	0.0063	0.0062
80.0000	17.0000	0.0060	0.0060	0.0059
80.0000	18.0000	0.0057	0.0058	0.0057
80.0000	19.0000	0.0054	0.0055	0 0054
80.0000	20.0000	0.0052	0.0052	0.0051
85.0000	0.0000	0.0143	0.0143	0 0143
85.0000	1.0000	0.0100	0 0101	0.0100
85.0000	2.0000	0.0096	0.0096	0 0095
85.0000	3.0000	0.0091	0.0092	0.0091
85.0000	4.0000	0 0087	0.0087	0 0086
85.0000	5.0000	0 0083	0.0083	0 0082
85.0000	6.0000	0.0079	0.0079	0.0078
85.0000	7.0000	0.0075	0.0076	0.0075
85.0000	8.0000	0.0072	0.0072	0.0071
85.0000	9.0000	0.0068	0.0069	0.0068
85.0000	10.0000	0.0065	0.0066	0.0065
85.0000	11.0000	0.0062	0 0063	0.0062
85.0000	12.0000	0.0059	0.0060	0.0059
85.0000	13.0000	0.0056	0 0057	0.0056
85.0000	14.0000	0.0054	0.0054	0.0053
85.0000	15.0000	0.0051	0.0052	0.0051
85.0000	16.0000	0.0049	0.0049	0.0049
85.0000	17.0000	0.0047	0.0047	0.0046
85.0000	18.0000	0.0044	0.0045	0.0044
85.0000	19.0000	0.0042	0.0043	0.0042
85.0000	20.0000	0.0040	0.0041	0.0040
90.0000	0.0000	0.0111	0.0111	0.0111
90.0000	1.0000	0.0078	0.0078	0.0078
90.0000	2.0000	0.0074	0.0075	0.0074
90.0000	3.0000	0.0071	0.0071	0.0070
90.0000	4.0000	0.0068	0.0068	0.0067
90.0000	5.0000	0.0064	0.0065	0.0064
90.0000	6.0000	0.0061	0.0062	0.0061
90.0000	7.0000	0.0059	0.0059	0.0058
90.0000	8.0000	0.0056	0.0056	0.0055
90.0000	9.0000	0.0053	0.0054	0.0053
90.0000	10.0000	0.0051	0.0051	0.0050
90.0000	11.0000	0.0048	0.0049	0.0048
90.0000	12.0000	0.0046	0.0046	0.0046
90.0000	13.0000	0.0044	0.0044	0.0044
90.0000	14.0000	0.0042	0.0042	0.0042
90.0000	15.0000	0.0040	0.0040	0.0040
90.0000	16.0000	0.0038	0.0038	0.0038
90.0000	17.0000	0.0036	0.0037	0.0036
90.0000	18.0000	0.0035	0.0035	0.0034
90.0000	19.0000	0.0033	0.0033	0.0033
90.0000	20.0000	0.0031	0.0032	0.0031
95.0000	0.0000	0.0087	0.0087	0.0087
95.0000	1.0000	0.0061	0.0061	0.0060
95.0000	2.0000	0.0058	0.0058	0.0058
95.0000	3.0000	0.0055	0.0056	0.0055
95.0000	4.0000	0.0053	0.0053	0.0052
95.0000	5.0000	0.0050	0.0051	0.0050
95.0000	6.0000	0.0048	0.0048	0.0048
95.0000	7.0000	0.0046	0.0046	0.0045
95.0000	8.0000	0.0044	0.0044	0.0043
95.0000	9.0000	0.0041	0.0042	0.0041

95.0000	10.0000	0.0040	0.0040	0.0039
95.0000	11.0000	0.0038	0.0038	0.0037
95.0000	12.0000	0.0036	0.0036	0.0036
95.0000	13.0000	0.0034	0.0035	0.0034
95.0000	14.0000	0.0033	0.0033	0.0032
95.0000	15.0000	0.0031	0.0031	0.0031
95.0000	16.0000	0.0030	0.0030	0.0029
95.0000	17.0000	0.0028	0.0029	0.0028
95.0000	18.0000	0.0027	0.0027	0.0027
95.0000	19.0000	0.0026	0.0026	0.0026
95.0000	20.0000	0.0025	0.0025	0.0024
100.0000	0.0000	0.0067	0.0067	0.0067
100.0000	1.0000	0.0047	0.0048	0.0047
100.0000	2.0000	0.0045	0.0045	0.0045
100.0000	3.0000	0.0043	0.0043	0.0043
100.0000	4.0000	0.0041	0.0041	0.0041
100.0000	5.0000	0.0039	0.0039	0.0039
100.0000	6.0000	0.0037	0.0038	0.0037
100.0000	7.0000	0.0036	0.0036	0.0035
100.0000	8.0000	0.0034	0.0034	0.0034
100.0000	9.0000	0.0032	0.0033	0.0032
100.0000	10.0000	0.0031	0.0031	0.0031
100.0000	11.0000	0.0029	0.0030	0.0029
100.0000	12.0000	0.0028	0.0028	0.0028
100.0000	13.0000	0.0027	0.0027	0.0027
100.0000	14.0000	0.0025	0.0026	0.0025
100.0000	15.0000	0.0024	0.0024	0.0024
100.0000	16.0000	0.0023	0.0023	0.0023
100.0000	17.0000	0.0022	0.0022	0.0022
100.0000	18.0000	0.0021	0.0021	0.0021
100.0000	19.0000	0.0020	0.0020	0.0020
100.0000	20.0000	0.0019	0.0019	0.0019

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TEST DATA NO    3

NUMBER OF LAYERS                        NL =     5
TOTAL SPACE STEPS OF ALL LAYERS         NX =   100
NUMBER OF TIME STEPS USED               NT =   100
SPACE STEPS FOR EACH PRINTOUT        NXOUT =    10
TIME STEPS FOR EACH PRINTOUT         NTOUT =    10
SPACE STEP-SIZE                         DX =     0.1000 (METER)
TIME STEP-SIZE                          DT =     0.1000 (YEAR)
COMBINED THICKNESS OF ALL LAYERS        XX =    10.0000 (METER)
TIME PERIOD SIMULATED                   TT =    10.0000 (YEAR)
INITIAL CONCENTRATION AT X=0            CO =     1.0000 (MASS/VOLUME)
DECAY COEFFICIENT FOR THE SOURCE         G =     0.0500 (1/YEAR)

LAYER   1,
EFFECTIVE DISPERSION COEFFICIENT         D =     1.0000 (SQUARE METER/YEAR)
EFFECTIVE PORE-WATER VELOCITY            V =     1.0000 (METER/YEAR)
EFFECTIVE TRANSFORMATION COEFF.          P =     0.1000 (1/YEAR)
THICKNESS OF THIS LAYER              THICK =     2.0000 (METER)
DISTANCE SIMULATED FOR THIS LAYER       XL =    15.0000 (METER)

LAYER   2,
EFFECTIVE DISPERSION COEFFICIENT         D =     2.0000 (SQUARE METER/YEAR)
EFFECTIVE PORE-WATER VELOCITY            V =     2.0000 (METER/YEAR)
EFFECTIVE TRANSFORMATION COEFF.          P =     0.1000 (1/YEAR)
THICKNESS OF THIS LAYER              THICK =     2.0000 (METER)
DISTANCE SIMULATED FOR THIS LAYER       XL =    30.0000 (METER)

LAYER   3,
EFFECTIVE DISPERSION COEFFICIENT         D =     3.0000 (SQUARE METER/YEAR)
EFFECTIVE PORE-WATER VELOCITY            V =     3.0000 (METER/YEAR)
EFFECTIVE TRANSFORMATION COEFF.          P =     0.1000 (1/YEAR)
THICKNESS OF THIS LAYER              THICK =     2.0000 (METER)
DISTANCE SIMULATED FOR THIS LAYER       XL =    45.0000 (METER)

LAYER   4,
EFFECTIVE DISPERSION COEFFICIENT         D =     4.0000 (SQUARE METER/YEAR)
EFFECTIVE PORE-WATER VELOCITY            V =     4.0000 (METER/YEAR)
EFFECTIVE TRANSFORMATION COEFF.          P =     0.1000 (1/YEAR)
THICKNESS OF THIS LAYER              THICK =     2.0000 (METER)
DISTANCE SIMULATED FOR THIS LAYER       XL =    60.0000 (METER)

LAYER   5,
EFFECTIVE DISPERSION COEFFICIENT         D =     5.0000 (SQUARE METER/YEAR)
EFFECTIVE PORE-WATER VELOCITY            V =     5.0000 (METER/YEAR)
EFFECTIVE TRANSFORMATION COEFF.          P =     0.1000 (1/YEAR)
THICKNESS OF THIS LAYER              THICK =     2.0000 (METER)
DISTANCE SIMULATED FOR THIS LAYER       XL =    75.0000 (METER)
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		CRANK-NICOLSON METHOD WITH		
DISTANCE	TIME	CENTERED	BACKWARD	FORWARD
0.0000	0.0000	1.0000	1.0000	1.0000
0.0000	1.0000	0.9512	0.9512	0.9512
0.0000	2.0000	0.9048	0.9048	0.9048
0.0000	3.0000	0.8607	0.8607	0.8607
0.0000	4.0000	0.8187	0.8187	0.8187
0.0000	5.0000	0.7788	0.7788	0.7788
0.0000	6.0000	0.7408	0.7408	0.7408
0.0000	7.0000	0.7047	0.7047	0.7047
0.0000	8.0000	0.6703	0.6703	0.6703
0.0000	9.0000	0.6376	0.6376	0.6376
0.0000	10.0000	0.6065	0.6065	0.6065
1.0000	0.0000	0.0000	0.0000	0.0000
1.0000	1.0000	0.6662	0.6696	0.6626
1.0000	2.0000	0.7676	0.7677	0.7675
1.0000	3.0000	0.7751	0.7746	0.7757
1.0000	4.0000	0.7567	0.7562	0.7573
1.0000	5.0000	0.7293	0.7289	0.7298
1.0000	6.0000	0.6987	0.6984	0.6991
1.0000	7.0000	0.6674	0.6671	0.6676
1.0000	8.0000	0.6364	0.6362	0.6365
1.0000	9.0000	0.6063	0.6062	0.6064
1.0000	10.0000	0.5772	0.5772	0.5773
2.0000	0.0000	0.0000	0.0000	0.0000
2.0000	1.0000	0.3365	0.3452	0.3274
2.0000	2.0000	0.5755	0.5786	0.5723
2.0000	3.0000	0.6561	0.6565	0.6557
2.0000	4.0000	0.6755	0.6751	0.6761
2.0000	5.0000	0.6690	0.6683	0.6697
2.0000	6.0000	0.6506	0.6500	0.6513
2.0000	7.0000	0.6270	0.6265	0.6275
2.0000	8.0000	0.6010	0.6007	0.6014
2.0000	9.0000	0.5745	0.5743	0.5748
2.0000	10.0000	0.5481	0.5480	0.5483
3.0000	0.0000	0.0000	0.0000	0.0000
3.0000	1.0000	0.1812	0.1900	0.1721
3.0000	2.0000	0.4586	0.4636	0.4534
3.0000	3.0000	0.5841	0.5854	0.5829
3.0000	4.0000	0.6292	0.6289	0.6295
3.0000	5.0000	0.6365	0.6358	0.6373
3.0000	6.0000	0.6259	0.6251	0.6267
3.0000	7.0000	0.6067	0.6061	0.6074
3.0000	8.0000	0.5836	0.5831	0.5841
3.0000	9.0000	0.5589	0.5586	0.5592
3.0000	10.0000	0.5339	0.5337	0.5341
4.0000	0.0000	0.0000	0.0000	0.0000
4.0000	1.0000	0.0800	0.0866	0.0734
4.0000	2.0000	0.3388	0.3455	0.3317
4.0000	3.0000	0.5014	0.5039	0.4987
4.0000	4.0000	0.5748	0.5750	0.5746
4.0000	5.0000	0.5992	0.5985	0.5999
4.0000	6.0000	0.5984	0.5976	0.5993
4.0000	7.0000	0.5851	0.5843	0.5858

4.0000	8.0000	0.5655	0.5649	0.5661
4.0000	9.0000	0.5431	0.5427	0.5435
4.0000	10.0000	0.5196	0.5194	0.5199
5.0000	0.0000	0.0000	0.0000	0.0000
5.0000	1.0000	0.0376	0.0419	0.0334
5.0000	2.0000	0.2591	0.2665	0.2512
5.0000	3.0000	0.4410	0.4445	0.4373
5.0000	4.0000	0.5349	0.5356	0.5342
5.0000	5.0000	0.5723	0.5718	0.5729
5.0000	6.0000	0.5792	0.5784	0.5801
5.0000	7.0000	0.5702	0.5694	0.5711
5.0000	8.0000	0.5533	0.5526	0.5539
5.0000	9.0000	0.5325	0.5320	0.5330
5.0000	10.0000	0.5102	0.5099	0.5105
6.0000	0.0000	0.0000	0.0000	0.0000
6.0000	1.0000	0.0155	0.0179	0.0133
6.0000	2.0000	0.1875	0.1951	0.1795
6.0000	3.0000	0.3777	0.3823	0.3728
6.0000	4.0000	0.4909	0.4922	0.4895
6.0000	5.0000	0.5427	0.5424	0.5430
6.0000	6.0000	0.5583	0.5575	0.5592
6.0000	7.0000	0.5545	0.5537	0.5554
6.0000	8.0000	0.5406	0.5399	0.5413
6.0000	9.0000	0.5217	0.5212	0.5223
6.0000	10.0000	0.5006	0.5003	0.5010
7.0000	0.0000	0.0000	0.0000	0.0000
7.0000	1.0000	0.0069	0.0082	0.0057
7.0000	2.0000	0.1392	0.1465	0.1318
7.0000	3.0000	0.3288	0.3342	0.3232
7.0000	4.0000	0.4557	0.4576	0.4537
7.0000	5.0000	0.5189	0.5189	0.5190
7.0000	6.0000	0.5418	0.5411	0.5426
7.0000	7.0000	0.5423	0.5415	0.5432
7.0000	8.0000	0.5309	0.5302	0.5317
7.0000	9.0000	0.5136	0.5131	0.5142
7.0000	10.0000	0.4935	0.4931	0.4939
8.0000	0.0000	0.0000	0.0000	0.0000
8.0000	1.0000	0.0028	0.0035	0.0023
8.0000	2.0000	0.0990	0.1055	0.0924
8.0000	3.0000	0.2803	0.2862	0.2740
8.0000	4.0000	0.4184	0.4209	0.4157
8.0000	5.0000	0.4933	0.4936	0.4931
8.0000	6.0000	0.5242	0.5236	0.5249
8.0000	7.0000	0.5295	0.5286	0.5304
8.0000	8.0000	0.5209	0.5202	0.5217
8.0000	9.0000	0.5053	0.5047	0.5059
8.0000	10.0000	0.4863	0.4859	0.4867
9.0000	0.0000	0.0000	0.0000	0.0000
9.0000	1.0000	0.0012	0.0016	0.0010
9.0000	2.0000	0.0720	0.0777	0.0663
9.0000	3.0000	0.2419	0.2483	0.2353
9.0000	4.0000	0.3873	0.3904	0.3841
9.0000	5.0000	0.4718	0.4724	0.4712
9.0000	6.0000	0.5094	0.5089	0.5100
9.0000	7.0000	0.5188	0.5180	0.5197
9.0000	8.0000	0.5127	0.5120	0.5136
9.0000	9.0000	0.4986	0.4980	0.4992
9.0000	10.0000	0.4805	0.4801	0.4809
10.0000	0.0000	0.0000	0.0000	0.0000
10.0000	1.0000	0.0005	0.0007	0.0004

10.0000	2.0000	0.0505	0.0553	0.0458
10.0000	3.0000	0.2052	0.2117	0.1983
10.0000	4.0000	0.3553	0.3590	0.3515
10.0000	5.0000	0.4490	0.4499	0.4480
10.0000	6.0000	0.4937	0.4934	0.4942
10.0000	7.0000	0.5077	0.5069	0.5086
10.0000	8.0000	0.5043	0.5035	0.5051
10.0000	9.0000	0.4918	0.4911	0.4924
10.0000	10.0000	0.4747	0.4742	0.4751

TEST DATA NO. 3 (CONTINUED)

TIME	DISTANCE	CRANK-NICOLSON METHOD WITH CENTERED	BACKWARD	FORWARD
0.0000	0.0000	1.0000	1.0000	1.0000
0.0000	1.0000	0.0000	0.0000	0.0000
0.0000	2.0000	0.0000	0.0000	0.0000
0.0000	3.0000	0.0000	0.0000	0.0000
0.0000	4.0000	0.0000	0.0000	0.0000
0.0000	5.0000	0.0000	0.0000	0.0000
0.0000	6.0000	0.0000	0.0000	0.0000
0.0000	7.0000	0.0000	0.0000	0.0000
0.0000	8.0000	0.0000	0.0000	0.0000
0.0000	9.0000	0.0000	0.0000	0.0000
0.0000	10.0000	0.0000	0.0000	0.0000
1.0000	0.0000	0.9512	0.9512	0.9512
1.0000	1.0000	0.6662	0.6696	0.6626
1.0000	2.0000	0.3365	0.3452	0.3274
1.0000	3.0000	0.1812	0.1900	0.1721
1.0000	4.0000	0.0800	0.0866	0.0734
1.0000	5.0000	0.0376	0.0419	0.0334
1.0000	6.0000	0.0155	0.0179	0.0133
1.0000	7.0000	0.0069	0.0082	0.0057
1.0000	8.0000	0.0028	0.0035	0.0023
1.0000	9.0000	0.0012	0.0016	0.0010
1.0000	10.0000	0.0005	0.0007	0.0004
2.0000	0.0000	0.9048	0.9048	0.9048
2.0000	1.0000	0.7676	0.7677	0.7675
2.0000	2.0000	0.5755	0.5786	0.5723
2.0000	3.0000	0.4586	0.4636	0.4534
2.0000	4.0000	0.3388	0.3455	0.3317
2.0000	5.0000	0.2591	0.2665	0.2512
2.0000	6.0000	0.1875	0.1951	0.1795
2.0000	7.0000	0.1392	0.1465	0.1318
2.0000	8.0000	0.0990	0.1055	0.0924
2.0000	9.0000	0.0720	0.0777	0.0663
2.0000	10.0000	0.0505	0.0553	0.0458
3.0000	0.0000	0.8607	0.8607	0.8607
3.0000	1.0000	0.7751	0.7746	0.7757
3.0000	2.0000	0.6561	0.6565	0.6557
3.0000	3.0000	0.5841	0.5854	0.5829
3.0000	4.0000	0.5014	0.5039	0.4987
3.0000	5.0000	0.4410	0.4445	0.4373
3.0000	6.0000	0.3777	0.3823	0.3728
3.0000	7.0000	0.3288	0.3342	0.3232
3.0000	8.0000	0.2803	0.2862	0.2740
3.0000	9.0000	0.2419	0.2483	0.2353
3.0000	10.0000	0.2052	0.2117	0.1983
4.0000	0.0000	0.8187	0.8187	0.8187
4.0000	1.0000	0.7567	0.7562	0.7573
4.0000	2.0000	0.6755	0.6751	0.6761
4.0000	3.0000	0.6292	0.6289	0.6295
4.0000	4.0000	0.5748	0.5750	0.5746

4.0000	5.0000	0.5349	0.5356	0.5342
4.0000	6.0000	0.4909	0.4922	0.4895
4.0000	7.0000	0.4557	0.4576	0.4537
4.0000	8.0000	0.4184	0.4209	0.4157
4.0000	9.0000	0.3873	0.3904	0.3841
4.0000	10.0000	0.3553	0.3590	0.3515
5.0000	0.0000	0.7788	0.7788	0.7788
5.0000	1.0000	0.7293	0.7289	0.7298
5.0000	2.0000	0.6690	0.6683	0.6697
5.0000	3.0000	0.6365	0.6358	0.6373
5.0000	4.0000	0.5992	0.5985	0.5999
5.0000	5.0000	0.5723	0.5718	0.5729
5.0000	6.0000	0.5427	0.5424	0.5430
5.0000	7.0000	0.5189	0.5189	0.5190
5.0000	8.0000	0.4933	0.4936	0.4931
5.0000	9.0000	0.4718	0.4724	0.4712
5.0000	10.0000	0.4490	0.4499	0.4480
6.0000	0.0000	0.7408	0.7408	0.7408
6.0000	1.0000	0.6987	0.6984	0.6991
6.0000	2 0000	0.6506	0.6500	0.6513
6.0000	3.0000	0.6259	0.6251	0.6267
6.0000	4.0000	0.5984	0.5976	0.5993
6.0000	5.0000	0.5792	0.5784	0.5801
6.0000	6.0000	0.5583	0.5575	0.5592
6.0000	7.0000	0.5418	0.5411	0.5426
6.0000	8.0000	0.5242	0.5236	0.5249
6.0000	9.0000	0.5094	0.5089	0.5100
6.0000	10.0000	0.4937	0.4934	0.4942
7.0000	0.0000	0.7047	0.7047	0.7047
7.0000	1.0000	0.6674	0.6671	0.6676
7.0000	2.0000	0.6270	0.6265	0.6275
7.0000	3.0000	0.6067	0.6061	0.6074
7.0000	4.0000	0.5851	0.5843	0.5858
7.0000	5.0000	0.5702	0.5694	0.5711
7.0000	6.0000	0.5545	0.5537	0.5554
7.0000	7.0000	0.5423	0.5415	0.5432
7.0000	8.0000	0.5295	0.5286	0.5304
7.0000	9.0000	0.5188	0.5180	0.5197
7.0000	10.0000	0.5077	0.5069	0.5086
8.0000	0.0000	0.6703	0.6703	0.6703
8.0000	1.0000	0.6364	0.6362	0.6365
8.0000	2.0000	0.6010	0.6007	0.6014
8.0000	3.0000	0.5836	0.5831	0.5841
8.0000	4.0000	0.5655	0.5649	0.5661
8.0000	5.0000	0.5533	0.5526	0.5539
8.0000	6.0000	0.5406	0.5399	0.5413
8.0000	7.0000	0.5309	0.5302	0.5317
8.0000	8.0000	0.5209	0.5202	0.5217
8.0000	9.0000	0.5127	0.5120	0.5136
8.0000	10.0000	0.5043	0.5035	0.5051
9.0000	0.0000	0.6376	0.6376	0.6376
9.0000	1.0000	0.6063	0.6062	0.6064
9.0000	2.0000	0.5745	0.5743	0.5748
9.0000	3.0000	0.5589	0.5586	0.5592
9.0000	4.0000	0.5431	0.5427	0.5435
9.0000	5.0000	0.5325	0.5320	0.5330
9.0000	6.0000	0.5217	0.5212	0.5223
9.0000	7.0000	0.5136	0.5131	0.5142
9.0000	8.0000	0.5053	0.5047	0.5059
9.0000	9.0000	0.4986	0.4980	0.4992

9.0000	10.0000	0.4918	0.4911	0.4924
10.0000	0.0000	0.6065	0.6065	0.6065
10.0000	1.0000	0.5772	0.5772	0.5773
10.0000	2.0000	0.5481	0.5480	0.5483
10.0000	3.0000	0.5339	0.5337	0.5341
10.0000	4.0000	0.5196	0.5194	0.5199
10.0000	5.0000	0.5102	0.5099	0.5105
10.0000	6.0000	0.5006	0.5003	0.5010
10.0000	7.0000	0.4935	0.4931	0.4939
10.0000	8.0000	0.4863	0.4859	0.4867
10.0000	9.0000	0.4805	0.4801	0.4809
10.0000	10.0000	0.4747	0.4742	0.4751

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TEST DATA NO.  4

NUMBER OF LAYERS                          NL =     5
TOTAL SPACE STEPS OF ALL LAYERS           NX =   100
NUMBER OF TIME STEPS USED                 NT =   100
SPACE STEPS FOR EACH PRINTOUT          NXOUT =    10
TIME STEPS FOR EACH PRINTOUT           NTOUT =    10
SPACE STEP-SIZE                           DX =     0.1000 (METER)
TIME STEP-SIZE                            DT =     0.1000 (YEAR)
COMBINED THICKNESS OF ALL LAYERS          XX =    10.0000 (METER)
TIME PERIOD SIMULATED                     TT =    10.0000 (YEAR)
INITIAL CONCENTRATION AT X=0              CO =     1.0000 (MASS/VOLUME)
DECAY COEFFICIENT FOR THE SOURCE           G =     0.0500 (1/YEAR)


LAYER  1.
EFFECTIVE DISPERSION COEFFICIENT           D =     5.0000 (SQUARE METER/YEAR)
EFFECTIVE PORE-WATER VELOCITY              V =     5.0000 (METER/YEAR)
EFFECTIVE TRANSFORMATION COEFF.            P =     0.1000 (1/YEAR)
THICKNESS OF THIS LAYER                THICK =     2.0000 (METER)
DISTANCE SIMULATED FOR THIS LAYER         XL =    75 0000 (METER)


LAYER  2.
EFFECTIVE DISPERSION COEFFICIENT           D =     4.0000 (SQUARE METER/YEAR)
EFFECTIVE PORE-WATER VELOCITY              V =     4.0000 (METER/YEAR)
EFFECTIVE TRANSFORMATION COEFF.            P =     0.1000 (1/YEAR)
THICKNESS OF THIS LAYER                THICK =     2 0000 (METER)
DISTANCE SIMULATED FOR THIS LAYER         XL =    60.0000 (METER)


LAYER  3.
EFFECTIVE DISPERSION COEFFICIENT           D =     3.0000 (SQUARE METER/YEAR)
EFFECTIVE PORE-WATER VELOCITY              V =     3.0000 (METER/YEAR)
EFFECTIVE TRANSFORMATION COEFF.            P =     0.1000 (1/YEAR)
THICKNESS OF THIS LAYER                THICK =     2.0000 (METER)
DISTANCE SIMULATED FOR THIS LAYER         XL =    45.0000 (METER)


LAYER  4.
EFFECTIVE DISPERSION COEFFICIENT           D =     2.0000 (SQUARE METER/YEAR)
EFFECTIVE PORE-WATER VELOCITY              V =     2.0000 (METER/YEAR)
EFFECTIVE TRANSFORMATION COEFF.            P =     0.1000 (1/YEAR)
THICKNESS OF THIS LAYER                THICK =     2.0000 (METER)
DISTANCE SIMULATED FOR THIS LAYER         XL =    30.0000 (METER)


LAYER  5.
EFFECTIVE DISPERSION COEFFICIENT           D =     1.0000 (SQUARE METER/YEAR)
EFFECTIVE PORE-WATER VELOCITY              V =     1.0000 (METER/YEAR)
EFFECTIVE TRANSFORMATION COEFF.            P =     0.1000 (1/YEAR)
THICKNESS OF THIS LAYER                THICK =     2.0000 (METER)
DISTANCE SIMULATED FOR THIS LAYER         XL =    15.0000 (METER)
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DISTANCE	TIME	CRANK-NICOLSON METHOD WITH CENTERED	BACKWARD	FORWARD
0.0000	0.0000	1.0000	1.0000	1.0000
0.0000	1.0000	0.9512	0.9512	0.9512
0.0000	2.0000	0.9048	0.9048	0.9048
0.0000	3.0000	0.8607	0.8607	0.8607
0.0000	4.0000	0.8187	0.8187	0.8187
0.0000	5.0000	0.7788	0.7788	0.7788
0.0000	6.0000	0.7408	0.7408	0.7408
0.0000	7.0000	0.7047	0.7047	0.7047
0.0000	8.0000	0.6703	0.6703	0.6703
0.0000	9.0000	0.6376	0.6376	0.6376
0.0000	10.0000	0.6065	0.6065	0.6065
1.0000	0.0000	0.0000	0.0000	0.0000
1.0000	1.0000	0.9109	0.9098	0.9121
1.0000	2.0000	0.8932	0.8925	0.8939
1.0000	3.0000	0.8530	0.8529	0.8530
1.0000	4.0000	0.8111	0.8112	0.8110
1.0000	5.0000	0.7711	0.7712	0.7711
1.0000	6.0000	0.7334	0.7334	0.7334
1.0000	7.0000	0.6976	0.6976	0.6976
1.0000	8.0000	0.6636	0.6636	0.6637
1.0000	9.0000	0.6313	0.6313	0.6313
1.0000	10.0000	0.6005	0.6005	0.6006
2.0000	0.0000	0.0000	0.0000	0.0000
2.0000	1.0000	0.8708	0.8695	0.8723
2.0000	2.0000	0.8792	0.8784	0.8799
2.0000	3.0000	0.8426	0.8424	0.8428
2.0000	4.0000	0.8025	0.8024	0.8025
2.0000	5.0000	0.7635	0.7635	0.7635
2.0000	6.0000	0.7263	0.7263	0.7263
2.0000	7.0000	0.6909	0.6909	0.6909
2.0000	8.0000	0.6572	0.6572	0.6572
2.0000	9.0000	0.6251	0.6251	0.6251
2.0000	10.0000	0.5946	0.5946	0.5946
3.0000	0.0000	0.0000	0.0000	0.0000
3.0000	1.0000	0.7687	0.7677	0.7698
3.0000	2.0000	0.8531	0.8518	0.8545
3.0000	3.0000	0.8292	0.8287	0.8296
3.0000	4.0000	0.7919	0.7918	0.7921
3.0000	5.0000	0.7540	0.7539	0.7540
3.0000	6.0000	0.7173	0.7173	0.7173
3.0000	7.0000	0.6824	0.6824	0.6824
3.0000	8.0000	0.6491	0.6491	0.6491
3.0000	9.0000	0.6175	0.6175	0.6174
3.0000	10.0000	0.5873	0.5873	0.5873
4.0000	0.0000	0.0000	0.0000	0.0000
4.0000	1.0000	0.6356	0.6369	0.6342
4.0000	2.0000	0.8165	0.8146	0.8185
4.0000	3.0000	0.8132	0.8124	0.8141
4.0000	4.0000	0.7809	0.7805	0.7812
4.0000	5.0000	0.7444	0.7443	0.7445
4.0000	6.0000	0.7084	0.7084	0.7085
4.0000	7.0000	0.6740	0.6740	0.6740

4.0000	8.0000	0.6411	0.6411	0.6411
4.0000	9.0000	0.6099	0.6099	0.6099
4.0000	10.0000	0.5801	0.5801	0.5801
5.0000	0.0000	0.0000	0.0000	0.0000
5.0000	1.0000	0.4443	0.4497	0.4386
5.0000	2.0000	0.7445	0.7427	0.7464
5.0000	3.0000	0.7835	0.7820	0.7850
5.0000	4.0000	0.7634	0.7627	0.7640
5.0000	5.0000	0.7308	0.7305	0.7311
5.0000	6.0000	0.6965	0.6964	0.6965
5.0000	7.0000	0.6629	0.6629	0.6629
5.0000	8.0000	0.6307	0.6307	0.6306
5.0000	9.0000	0.5999	0.6000	0.5999
5.0000	10.0000	0.5707	0.5707	0.5707
6.0000	0.0000	0.0000	0.0000	0.0000
6.0000	1.0000	0.2724	0.2803	0.2640
6.0000	2.0000	0.6518	0.6512	0.6525
6.0000	3.0000	0.7441	0.7422	0.7461
6.0000	4.0000	0.7425	0.7414	0.7436
6.0000	5.0000	0.7162	0.7157	0.7167
6.0000	6.0000	0.6842	0.6840	0.6844
6.0000	7.0000	0.6518	0.6517	0.6518
6.0000	8.0000	0.6203	0.6203	0.6203
6.0000	9.0000	0.5901	0.5902	0.5901
6.0000	10.0000	0.5614	0.5614	0.5614
7.0000	0.0000	0.0000	0.0000	0.0000
7.0000	1.0000	0.1150	0.1223	0.1076
7.0000	2.0000	0.4944	0.4971	0.4916
7.0000	3.0000	0.6617	0.6602	0.6633
7.0000	4.0000	0.6984	0.6968	0.7001
7.0000	5.0000	0.6884	0.6874	0.6894
7.0000	6.0000	0.6634	0.6629	0.6640
7.0000	7.0000	0.6343	0.6340	0.6345
7.0000	8.0000	0.6046	0.6045	0.6047
7.0000	9.0000	0.5756	0.5755	0.5756
7.0000	10.0000	0.5477	0.5477	0.5477
8.0000	0.0000	0.0000	0.0000	0.0000
8.0000	1.0000	0.0392	0.0434	0.0350
8.0000	2.0000	0.3404	0.3461	0.3343
8.0000	3.0000	0.5611	0.5612	0.5610
8.0000	4.0000	0.6417	0.6401	0.6434
8.0000	5.0000	0.6541	0.6527	0.6556
8.0000	6.0000	0.6397	0.6388	0.6406
8.0000	7.0000	0.6156	0.6151	0.6160
8.0000	8.0000	0.5885	0.5882	0.5887
8.0000	9.0000	0.5610	0.5609	0.5611
8.0000	10.0000	0.5342	0.5341	0.5342
9.0000	0.0000	0.0000	0.0000	0.0000
9.0000	1.0000	0.0053	0.0063	0.0043
9.0000	2.0000	0.1475	0.1547	0.1401
9.0000	3.0000	0.3655	0.3698	0.3609
9.0000	4.0000	0.5014	0.5021	0.5008
9.0000	5.0000	0.5597	0.5589	0.5607
9.0000	6.0000	0.5741	0.5730	0.5752
9.0000	7.0000	0.5668	0.5659	0.5678
9.0000	8.0000	0.5497	0.5491	0.5503
9.0000	9.0000	0.5283	0.5279	0.5288
9.0000	10.0000	0.5055	0.5053	0.5057
10.0000	0.0000	0.0000	0.0000	0.0000
10.0000	1.0000	0.0005	0.0007	0.0004

10.0000	2.0000	0.0505	0.0553	0.0458
10.0000	3.0000	0.2052	0.2117	0.1983
10.0000	4.0000	0.3553	0.3590	0.3515
10.0000	5.0000	0.4490	0.4499	0.4480
10.0000	6.0000	0.4937	0.4934	0.4942
10.0000	7.0000	0.5077	0.5069	0.5086
10.0000	8.0000	0.5043	0.5035	0.5052
10.0000	9.0000	0.4918	0.4912	0.4925
10.0000	10.0000	0.4747	0.4743	0.4752

TEST DATA NO. 4 (CONTINUED)

TIME	DISTANCE	CRANK-NICOLSON METHOD WITH CENTERED	BACKWARD	FORWARD
0.0000	0.0000	1.0000	1.0000	1.0000
0.0000	1.0000	0.0000	0.0000	0.0000
0.0000	2.0000	0.0000	0.0000	0.0000
0.0000	3.0000	0.0000	0.0000	0.0000
0.0000	4.0000	0.0000	0.0000	0.0000
0.0000	5.0000	0.0000	0.0000	0.0000
0.0000	6.0000	0.0000	0.0000	0.0000
0.0000	7.0000	0.0000	0.0000	0.0000
0.0000	8.0000	0.0000	0.0000	0.0000
0.0000	9.0000	0.0000	0.0000	0.0000
0.0000	10.0000	0.0000	0.0000	0.0000
1.0000	0.0000	0.9512	0.9512	0.9512
1.0000	1.0000	0.9109	0.9098	0.9121
1.0000	2.0000	0.8708	0.8695	0.8723
1.0000	3.0000	0.7687	0.7677	0.7698
1.0000	4.0000	0.6356	0.6369	0.6342
1.0000	5.0000	0.4443	0.4497	0.4386
1.0000	6.0000	0.2724	0.2803	0.2640
1.0000	7.0000	0.1150	0.1223	0.1076
1.0000	8.0000	0.0392	0.0434	0.0350
1.0000	9.0000	0.0053	0.0063	0.0043
1.0000	10.0000	0.0005	0.0007	0.0004
2.0000	0.0000	0.9048	0.9048	0.9048
2.0000	1.0000	0.8932	0.8925	0.8939
2.0000	2.0000	0.8792	0.8784	0.8799
2.0000	3.0000	0.8531	0.8518	0.8545
2.0000	4.0000	0.8165	0.8146	0.8185
2.0000	5.0000	0.7445	0.7427	0.7464
2.0000	6.0000	0.6518	0.6512	0.6525
2.0000	7.0000	0.4944	0.4971	0.4916
2.0000	8.0000	0.3404	0.3461	0.3343
2.0000	9.0000	0.1475	0.1547	0.1401
2.0000	10.0000	0.0505	0.0553	0.0458
3.0000	0.0000	0.8607	0.8607	0.8607
3.0000	1.0000	0.8530	0.8529	0.8530
3.0000	2.0000	0.8426	0.8424	0.8428
3.0000	3.0000	0.8292	0.8287	0.8296
3.0000	4.0000	0.8132	0.8124	0.8141
3.0000	5.0000	0.7835	0.7820	0.7850
3.0000	6.0000	0.7441	0.7422	0.7461
3.0000	7.0000	0.6617	0.6602	0.6633
3.0000	8.0000	0.5611	0.5612	0.5610
3.0000	9.0000	0.3655	0.3698	0.3609
3.0000	10.0000	0.2052	0.2117	0.1983
4.0000	0.0000	0.8187	0.8187	0.8187
4.0000	1.0000	0.8111	0.8112	0.8110
4.0000	2.0000	0.8025	0.8024	0.8025
4.0000	3.0000	0.7919	0.7918	0.7921
4.0000	4.0000	0.7809	0.7805	0.7812

4.0000	5.0000	0.7634	0.7627	0.7640
4.0000	6.0000	0.7425	0.7414	0.7436
4.0000	7.0000	0.6984	0.6968	0.7001
4.0000	8.0000	0.6417	0.6401	0.6434
4.0000	9.0000	0.5014	0.5021	0.5008
4.0000	10.0000	0.3553	0.3590	0.3515
5.0000	0.0000	0.7788	0.7788	0.7788
5.0000	1.0000	0.7711	0.7712	0.7711
5.0000	2.0000	0.7635	0.7635	0.7635
5.0000	3.0000	0.7540	0.7539	0.7540
5.0000	4.0000	0.7444	0.7443	0.7445
5.0000	5.0000	0.7308	0.7305	0.7311
5.0000	6.0000	0.7162	0.7157	0.7167
5.0000	7.0000	0.6884	0.6874	0.6894
5.0000	8.0000	0.6541	0.6527	0.6556
5.0000	9.0000	0.5597	0.5589	0.5607
5.0000	10.0000	0.4490	0.4499	0.4480
6.0000	0.0000	0.7408	0.7408	0.7408
6.0000	1.0000	0.7334	0.7334	0.7334
6.0000	2.0000	0.7263	0.7263	0.7263
6.0000	3.0000	0.7173	0.7173	0.7173
6.0000	4.0000	0.7084	0.7084	0.7085
6.0000	5.0000	0.6965	0.6964	0.6965
6.0000	6.0000	0.6842	0.6840	0.6844
6.0000	7.0000	0.6634	0.6629	0.6640
6.0000	8.0000	0.6397	0.6388	0.6406
6.0000	9.0000	0.5741	0.5730	0.5752
6.0000	10.0000	0.4937	0.4934	0.4942
7.0000	0.0000	0.7047	0.7047	0.7047
7.0000	1.0000	0.6976	0.6976	0.6976
7.0000	2.0000	0.6909	0.6909	0.6909
7.0000	3.0000	0.6824	0.6824	0.6824
7.0000	4.0000	0.6740	0.6740	0.6740
7.0000	5.0000	0.6629	0.6629	0.6629
7.0000	6.0000	0.6518	0.6517	0.6518
7.0000	7.0000	0.6343	0.6340	0.6345
7.0000	8.0000	0.6156	0.6151	0.6160
7.0000	9.0000	0.5668	0.5659	0.5678
7.0000	10.0000	0.5077	0.5069	0.5086
8.0000	0.0000	0.6703	0.6703	0.6703
8.0000	1.0000	0.6636	0.6636	0.6637
8.0000	2.0000	0.6572	0.6572	0.6572
8.0000	3.0000	0.6491	0.6491	0.6491
8.0000	4.0000	0.6411	0.6411	0.6411
8.0000	5.0000	0.6307	0.6307	0.6306
8.0000	6.0000	0.6203	0.6203	0.6203
8.0000	7.0000	0.6046	0.6045	0.6047
8.0000	8.0000	0.5885	0.5882	0.5887
8.0000	9.0000	0.5497	0.5491	0.5503
8.0000	10.0000	0.5043	0.5035	0.5052
9.0000	0.0000	0.6376	0.6376	0.6376
9.0000	1.0000	0.6313	0.6313	0.6313
9.0000	2.0000	0.6251	0.6251	0.6251
9.0000	3.0000	0.6175	0.6175	0.6174
9.0000	4.0000	0.6099	0.6099	0.6099
9.0000	5.0000	0.5999	0.6000	0.5999
9.0000	6.0000	0.5901	0.5902	0.5901
9.0000	7.0000	0.5756	0.5755	0.5756
9.0000	8.0000	0.5610	0.5609	0.5611
9.0000	9.0000	0.5283	0.5279	0.5288

9.0000	10.0000	0.4918	0.4912	0 4925
10.0000	0.0000	0.6065	0.6065	0.6065
10.0000	1.0000	0.6005	0.6005	0.6006
10.0000	2.0000	0.5946	0.5946	0.5946
10.0000	3.0000	0.5873	0.5873	0.5873
10.0000	4.0000	0.5801	0.5801	0.5801
10.0000	5.0000	0.5707	0.5707	0.5707
10.0000	6.0000	0.5614	0.5614	0.5614
10.0000	7.0000	0.5477	0.5477	0.5477
10.0000	8.0000	0.5342	0.5341	0.5342
10.0000	9.0000	0.5055	0.5053	0.5057
10.0000	10.0000	0.4747	0.4743	0.4752

VITA

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